

Scaling the near-wall axial turbulent stress in the zero pressure gradient boundary layer

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Based upon high resolution LDA measurements over a range of momentum deficit thickness Reynolds numbers ($R_\theta = U_\infty \theta / \nu$) from 1430 to 31 000, DeGraaff and Eaton [J. Fluid Mech. **422**, 319 (2000)] propose a new mixed scaling for the near-wall region profile of the axial turbulent stress, $\overline{u^2}$. The present results support the validity of this scaling over an extended Reynolds number range $1000 \leq R_\theta \leq 5 \times 10^6$. © 2001 American Institute of Physics. [DOI: 10.1063/1.1368852]

I. INTRODUCTION

Despite many years of effort, controversy still exists regarding the scaling behavior of the axial turbulent stress $\overline{u^2}$ in the near-wall region of the turbulent boundary layer. The main technical reason for this relates to the inability of measurement sensors to spatially resolve the turbulent intensity produced by small scale, near-wall motions. In particular, as the Reynolds number increases, the smallest scales of motion become diminishingly small relative to the largest scales. Therefore, in any given facility, a fixed-sized probe will experience a decrease in spatial resolution as the Reynolds number increases. Despite the clear evidence of attenuation in $\overline{u^2}$ resulting from insufficient spatial resolution,²⁻⁶ the scaling issue of $\overline{u^2}$ remains clouded in the literature primarily from a failure to accurately account for the competing effects of Reynolds number and probe spatial resolution on the data.

Regarding imperfect spatial resolution effects, Blackwelder and Haritonidis² note that the peak value of $\overline{u^{2+}}$ ($\equiv \overline{u^2}/u_\tau^2$) decreases with increasing Reynolds number when probe scales exceed more than about 20 viscous units, where u_τ represents the friction velocity. A similar trend exists in the data of Purtell *et al.*⁷ Johansson and Alfredsson³ explore this issue explicitly and show that the peak in $\overline{u^{2+}}$ evinces attenuation for inner normalized probe scales between $4 < l^+ < 14$. Based upon a study involving multiple facilities and measurement techniques, Alfredsson *et al.*⁴ determined that probe scales less than about 6 viscous units are required to capture the value $u'/U = 0.4$ in the viscous sublayer, where $u' \equiv \sqrt{\overline{u^2}}$. Similarly, based upon high resolution hot-wire measurements over the range $1010 \leq R_\theta \leq 4850$, Klewicki and Falco⁵ estimate that probe scales less than about 8 viscous units are required to capture the proper Reynolds number dependence in the peak value of $\overline{u^{2+}}$. More recently, Metzger and Klewicki,⁸ selecting only high resolution boundary layer data ($l^+ \leq 10$), demonstrate a logarithmic

dependence of the peak value of u'^+ on R_θ over the range $1000 \leq R_\theta \leq 5 \times 10^6$.

DeGraaff and Eaton¹ were able to maintain a probe dimension of $l^+ \leq 10$ over a substantial Reynolds number range ($1430 \leq R_\theta \leq 31\,000$) and thus, have considerably clarified the Reynolds number dependence of $\overline{u^2}$. Specifically, they show that sublayer and buffer layer profiles of $\overline{u^2}$ at different R_θ fall on a single curve when plotted as $\overline{u^2}/(u_\tau U_\infty)$ versus y^+ ($\equiv y u_\tau / \nu$). This paper provides evidence that the mixed scaling proposed by DeGraaff and Eaton holds over a much larger Reynolds number range, $1000 \leq R_\theta \leq 5 \times 10^6$.

II. FACILITIES AND EXPERIMENTS

The present work draws primarily upon the high resolution experimental data sets of Klewicki and Falco,⁵ DeGraaff and Eaton,¹ and Metzger and Klewicki.⁸ The distinguishing feature of these experiments is that, for all Reynolds numbers considered, the inner normalized probe scale is less than 10. DeGraaff and Eaton present two component LDA measurements in a laboratory wind tunnel enclosed within a pressure vessel. Hot-wire anemometry is utilized in the studies of Klewicki and Falco (spanwise vorticity probe) and Metzger and Klewicki (five wire rake). The former were acquired in a low speed wind tunnel, while the later were taken in the atmospheric surface layer at the SLTEST site located in Utah's western desert.

In order to recast the data of Metzger and Klewicki in terms of the mixed scaling proposed by DeGraaff and Eaton, an estimate of u_τ/U_∞ at the SLTEST site was required. This ratio was obtained using simultaneous measurements from a minisodar (Aerovironment model 4000) and a 2.4 m diam floating element, surface shear force drag plate. Details regarding the drag plate are presented by Sadr and Klewicki.⁹ The minisodar is capable of acquiring three-component mean and fluctuating velocity data, 15–200 m above ground level at 5 m intervals. The manufacturer specifies an accuracy of ± 0.25 m/s in the horizontal wind speed. Figure 1 presents the inner normalized mean horizontal wind profile from 100 min of minisodar data recorded through sunset on 11 August

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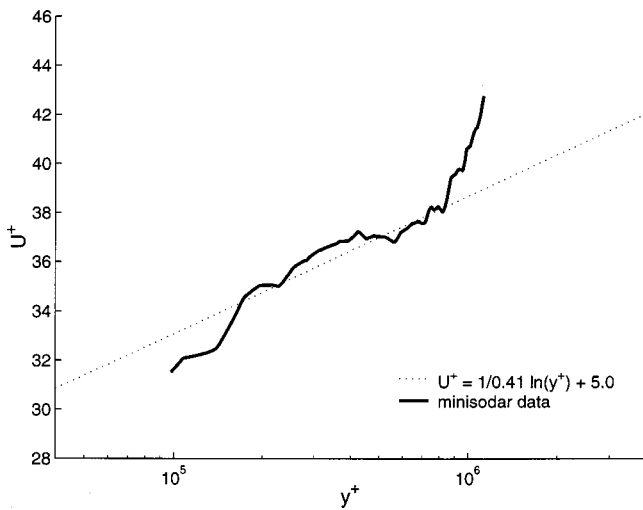


FIG. 1. Inner normalized horizontal wind profile from the minisodar averaged over 100 min. The dashed line represents Coles (Ref. 10) log law.

1999. Concurrent temperature flux measurements, using a pair of sonic anemometers positioned at a height of 2 m, reveal that near neutral stability conditions existed during these 100 min of minisodar data. The stability parameter y/L , with L denoting the Monin–Obukhov length, increased slowly from -0.2 to 0.4 during this time.

As apparent in Fig. 1, the average minisodar profile follows remarkably close to Coles¹⁰ log law line. The data depart from Coles line near $y^+ = 8.3 \times 10^5$. This departure is generally associated with an abrupt change in flow speed and direction that delineates the top of the surface layer, the region of the atmospheric boundary layer where mechanical shear dominates the turbulence. For this reason, δ is chosen as the height above the surface at which the mean horizontal wind data depart from a logarithmic trend. Interestingly, a more comprehensive examination of minisodar data indicates that for the range of typical wind speeds at the SLTEST site the inner normalized surface layer thickness, δ^+ , does not vary appreciably. From the present data, $\delta^+ = 8.3 \times 10^6$, which yields a value of 38.3 for U_∞/u_τ . Note the 100 min time average used to compute Fig. 1 represents 285 integral time scales based on the above estimate of δ/U_∞ .

Two points need to be made regarding the procedure for estimating δ . First of all, the data of DeGraaff and Eaton show that the mean profile wake parameter, $\Delta U/u_\tau$, decreases with R_θ for $R_\theta \geq 5000$. Thus, the absence of the wake effect from the above estimate of δ is likely to be of minor consequence. For example, choosing a value $\Delta U/u_\tau = 2.15$, corresponding to the measurement of DeGraaff and Eaton at $R_\theta = 31\,000$ (a significant overestimate for the data of Metzger and Klewicki), results in only about a 5% difference in U_∞/u_τ . Second, the estimate of δ^+ would have to be reduced by 4.5×10^5 or increased by 1.0×10^6 to yield a net $\pm 5\%$ variation in the estimate of u_τ/U_∞ . Thus, given the measurement capabilities, the stated value for U_∞/u_τ is highly insensitive to the relevant sources of uncertainty.

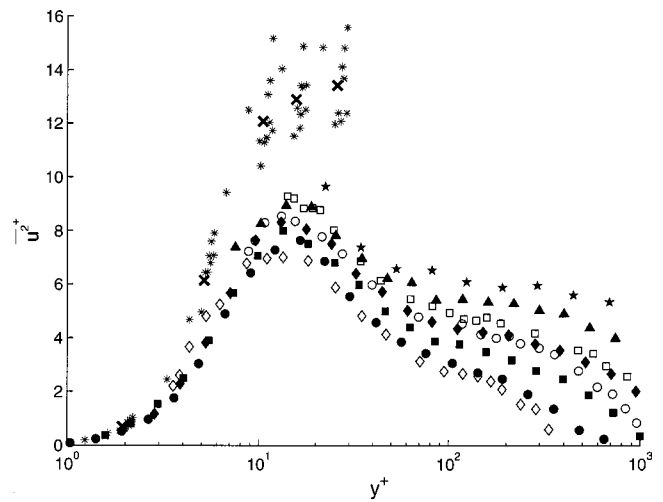


FIG. 2. $\overline{u^2}$ normalized by inner coordinates. Klewicki and Falco (Ref. 5): (\diamond) $R_\theta = 1010$; (\circ) $R_\theta = 2870$; (\square) $R_\theta = 4850$. DeGraaff and Eaton (Ref. 1): (\bullet) $R_\theta = 1430$; (\blacksquare) $R_\theta = 2900$; (\blacklozenge) $R_\theta = 5200$; (\blacktriangle) $R_\theta = 13\,000$; (\ast) $R_\theta = 31\,000$. Metzger and Klewicki (Ref. 8): (\ast) $R_\theta = 5 \times 10^6$. \times represents an average over the ten independent runs from Metzger and Klewicki.

III. RESULTS AND DISCUSSION

Figure 2 presents inner normalized $\overline{u^2}$ data from Klewicki and Falco⁵ ($1010 \leq R_\theta \leq 4850$), DeGraaff and Eaton¹ ($1430 \leq R_\theta \leq 31\,000$), and Metzger and Klewicki⁸ ($R_\theta \approx 5 \times 10^6$). Regarding the latter, 10 independent, 5-min-long, runs were acquired from a five wire rake spanning $2 < y^+ < 30$. The individual 5 min averages are plotted as gray asterisk with the composite 50 min averages shown as \times . Note, the apparent scatter in the 5 min average data does not necessarily represent the uncertainty of the data but arises due to the fact that none of the individual runs has achieved statistical convergence. The data presented in this way, however, do show that for $y^+ > 5.5$ each of the 5 min average profiles remains clearly distinct from the lower R_θ

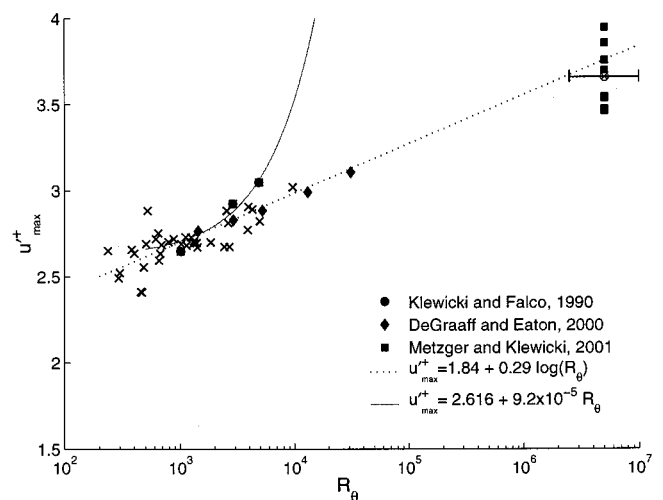


FIG. 3. Peak value of u'^+ as a function of Reynolds number. \times denotes composite low Reynolds number data selected from the available literature with the criterion that $l^+ \leq 10$ (see Metzger and Klewicki—Ref. 8—for references). \otimes represents the average value of the combined data from Metzger and Klewicki.

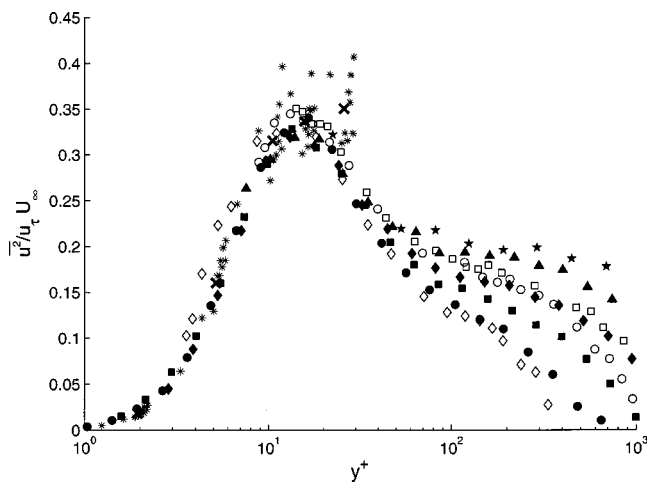


FIG. 4. $\overline{u'^2}$ normalized by mixed coordinates. Symbols as in Fig. 2.

results. From Fig. 2, one may also observe the exceptional agreement between the data of Klewicki and Falco and DeGraaff and Eaton, acquired at similar R_θ . Overall, the composite data exhibit a significant and unambiguous Reynolds number dependence.

Figure 3 displays the Reynolds number dependence of the peak value of u'^+ . Included are the data from Fig. 2 as well as the peak values selected from other boundary layer data sets in the literature for which $l^+ \leq 10$ (see Metzger and Klewicki⁸ for the list of references). The linear and logarithmic curve fits of Klewicki and Falco and Metzger and Klewicki, respectively, are also plotted. The linear fit adequately describes the trend in the peak value of u'^+ over its intended range of applicability ($R_\theta \leq 4000$). The most significant feature to note, however, is that the data of DeGraaff and Eaton rest directly on top of the logarithmic curve fit. Importantly, the logarithmic curve fit was derived solely from the low R_θ data (denoted by the symbol \times in Fig. 3), independent of the data of DeGraaff and Eaton and that of Metzger and Klewicki. Therefore, when properly selected according to sufficient spatial resolution, data from the exist-

ing literature may be extrapolated to *predict*, to a very high degree of precision, the trend indicated by the high Reynolds number results in Fig. 3.

Figure 4 shows the $\overline{u'^2}^+$ data of Fig. 2 multiplied by u_τ/U_∞ . Clearly, the near-wall data fall convincingly onto a single universal curve with a peak value of about 0.33 ± 0.01 . The results of Figs. 3 and 4 indicate that when probes having inner normalized dimension less than 10 are employed, the mixed scaling proposed by DeGraaff and Eaton holds over the range $1000 \leq R_\theta \leq 5 \times 10^6$.

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