Decomposition and Visualization of Soil Stiffness Tensors with VEES

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We present a decomposition to reduce the number of components of fourth-order elastic-plastic stiffness tensors to second-order 3x3 tensors for easier visualization.

**Approach**

Polar decomposition, followed by eigen-decomposition on the polar "stretch".

If any resulting eigenvalue is significantly lower than the others, the material is less stiff in that eigen-direction.

The associated second-order eigentensor represents the mode of stress to which the material becomes most vulnerable.

**Limitations**

- Small Deformation Theory
- Minor symmetry in tensors
- No elastic-plastic coupling

**Stage One: Polar decomposition**

Uniquely separates a matrix into two components:

\[ E = RS \]

- \( S \): stretch (symmetric positive-semidefinite matrix)
- \( R \): rotation (orthonormal)

For non-associated materials, we conjecture the polar rotation \( R \) quantifies misalignment of the yield surface normal and the plastic potential surface normal.

**Stage Two: Eigen-decomposition**

- Reduced eigenvalue: solid is less stiff.
- Visualize associated second-order eigentensor.

**Experiment**

- **Stage 1**: Self-weight compression (–Z)
- **Stage 2**: Two point loads: –Z component (0.9659 kN) and +X component (1294 kN)

Eigentensor glyphs colored by lowest eigenvalue, scaled across all time steps. Black cubes indicate singularity (negative stiffness eigenvalues).

Drucker-Prager: associated, elastic perfectly-plastic.

Failed with tension behind point loads, shear around, and triaxial compression in front. This is more evident with the Lode angle color scheme.

Stage 1 induced hardening at the bottom, with some singularity. Stage

**Visualization**

Tensor glyphs were drawn by stretching a unit sphere according to the formula

\[ n = \frac{\sigma_{ij} n_i n_j}{\sigma_{ii}} \]

\( n \) is a unit length direction vector from the center of the sphere to a point on the surface.

**Associated Scalar Fields For Color**

1. **Lowest Eigenvalue**: highlights loss of stiffness
2. **Lode Angle**: categorizes deviatoric mode for stress tensors

\[ \theta = \frac{1}{2} \sin^{-1} \left( \frac{J_2}{J_1} \right)^{1/2} \]

Where \( J_2 = \frac{1}{2} \| \sigma \| \) and \( J_1 = \text{det} S \) is the deviatoric part of the stress tensor

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Beta program (without stiffness visualization)

http://neesforge.nees.org/projects/vees/