ME 2000 Economics Handout

Simple Interest:

$I = P\,i\,N$ or $F = P + P\,i\,N$

Compound Interest:

$F/P = (1 + i)^N$

Effective Interest Rate:

$$i = \left(1 + \frac{r}{m}\right)^m - 1$$ continuous compounding: $i = e^r - 1$

where

i is effective interest rate, r is nominal interest rate, and m is compounding periods per year

Other Equations:

Present value given future value: $$P/F = \frac{1}{(1+i)^N}$$

Annual payment given future value: $$A/F = \frac{i}{(1+i)^N - 1}$$

Future value of annual payments: $$F/A = \frac{(1+i)^N - 1}{i}$$

Annual payment given present value: $$A/P = \frac{i(1+i)^N}{(1+i)^N - 1}$$

Present value of annual payments: $$P/A = \frac{(1+i)^N - 1}{i(1+i)^N}$$

Examples:

1. $10,000 invested at 9% interest compounded yearly will be worth how much after 7 years?

$$F = P\,(1 + i)^N = 10{,}000(1 + 0.09)^7$$

2. I need to pay $40,000 in 10 years for college tuition. How much do I need to invest today at 13% interest to realize this goal?

$$P = F\frac{1}{(1+i)^N} = \frac{40{,}000}{(1+0.13)^{10}}$$

3. I need to pay $40,000 in 10 years for tuition. How much do I need to invest per month at 12% interest to realize this goal?

Trick: 12% per year is 1% per month

$$A = F\frac{i}{(1+i)^N - 1} = 40{,}000\frac{.01}{(1.01)^{120} - 1}$$

4. If I save $2,000/yr. at 6% interest, how much will I have in 8 years?

$$F = A\frac{(1+i)^N - 1}{i} = 2{,}000\frac{(1.06)^8 - 1}{0.06}$$

5. I want to borrow $15,000 to buy a car. The interest rate is 6%. What will my monthly payment be on a 5 year loan?

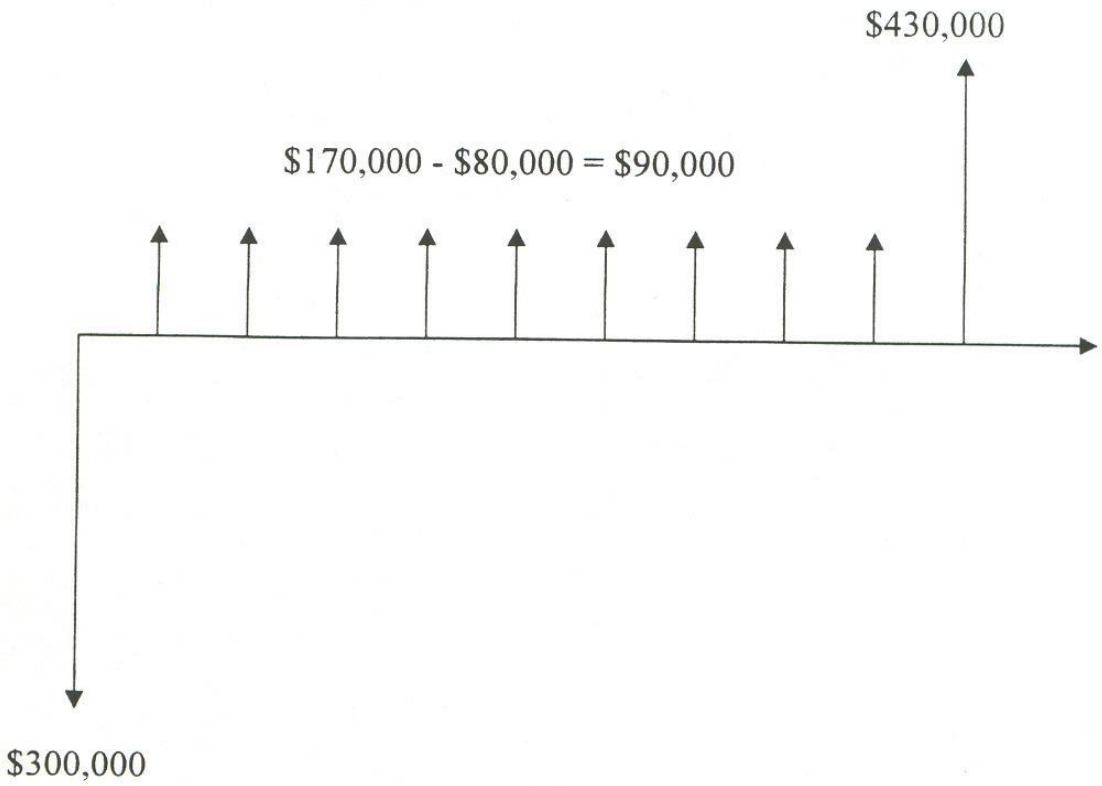
$$A = P\frac{i(1+i)^N}{(1+i)^N - 1} = 15{,}000\frac{.005(1.005)^{60}}{(1.005)^{60} - 1}$$

6. My budget allows $175/mo. for a car. How much can I spend on the car?

$$P = A\frac{(1+i)^N - 1}{i(1+i)^N} = 175\frac{(1.01)^{60} - 1}{0.01(1.01)^{60}}$$

Evaluating a Financial Opportunity:

I have the opportunity to buy a metal casting facility for $300,000. The facility requires$80,000/yr. in salaries, maintenance, insurance, etc. At the end of 10 years I will sell it for $340,000. The facility produces annual revenue of $170,000. As an alternative, I could invest my $300,000 in long term bonds at an interest rate of 10% compounded annually. What should I do?



Let's move all the money to the present (Present Worth Analysis):

$$P = -300,000 + 90,000(P/A, 10\%, 10) + 340,000(P/F, 10\%, 10)$$

or

$$P = -300,000 + 90,000(P/A, 10\%, 9) + 430,000(P/F, 10\%, 10)$$

Let's move all the money to the end of the 10 yrs. (Future Worth Analysis):

$$F = -300,000(F/P, 10\%, 10) + 90,000(F/A, 10\%, 10) + 340,000$$

Lets analyze this based on annual payments (Annual Worth Analysis):

$$A = -300,000(A/P, 10\%, 10) + 90,000 + 340,000(A/F, 10\%, 10)$$

What is the actual yearly operating cost for your car?

Price:	\$15,000
Maint., Gas, Insurance, …:	\$1,440
Life:	5 yrs.
Salvage:	\$5,000
Current interest rate:	6%

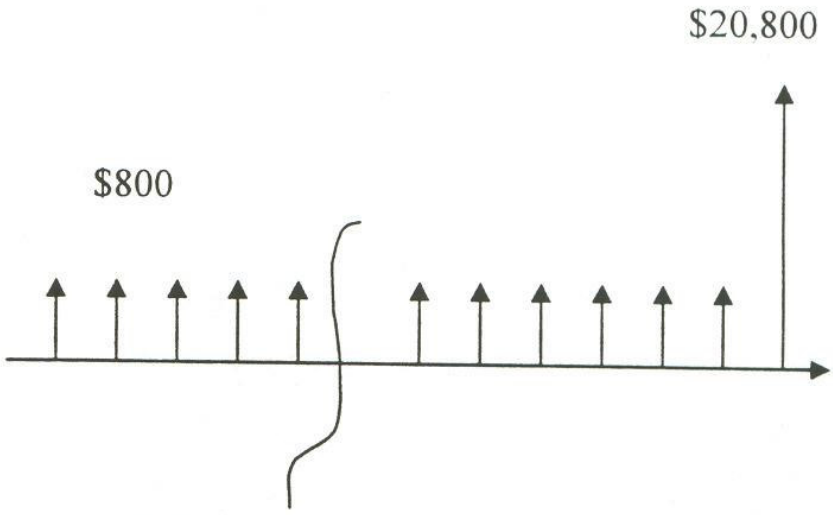
$$A = 15{,}000(A/P, 6\%, 5) + 1{,}440 + 5{,}000(A/F, 6\%, 5)$$

Monthly?

$$A = 15{,}000(A/P, 0.5\%, 60) + 120 + 5{,}000(A/F, 0.5\%, 60)$$

Why aren't these the same?

Four years ago I bought a Municipal Bond. I paid \$20,000. The bond pays me 4% of the note value every 6 months for 20 years. At the end of the 20 years, I get my 20,000 back. I have decided I would like to sell the bond. The prevailing interest rate is 5%. What is the current value of the bond?



$$P = 800(P/A, 5\%, 16) + 20{,}000(P/F, 5\%, 16)$$

Break Even Analysis

In my factory, I currently produce castings using manual sand casting. I have workers who make molds one at a time, and each mold produces one part and then is destroyed. An alternative has been presented which is Die Casting. Using die casting, a steel mold is produced that will automatically produce high volumes of castings very rapidly. Should I switch processes?

Here are the numbers:

Sand Casting:

- Equipment: $5,000 x 5 stations
- mold materials: $5 per part
- foundry metal: $3
- Production Rate: 2 parts per hour per station

Die Casting:

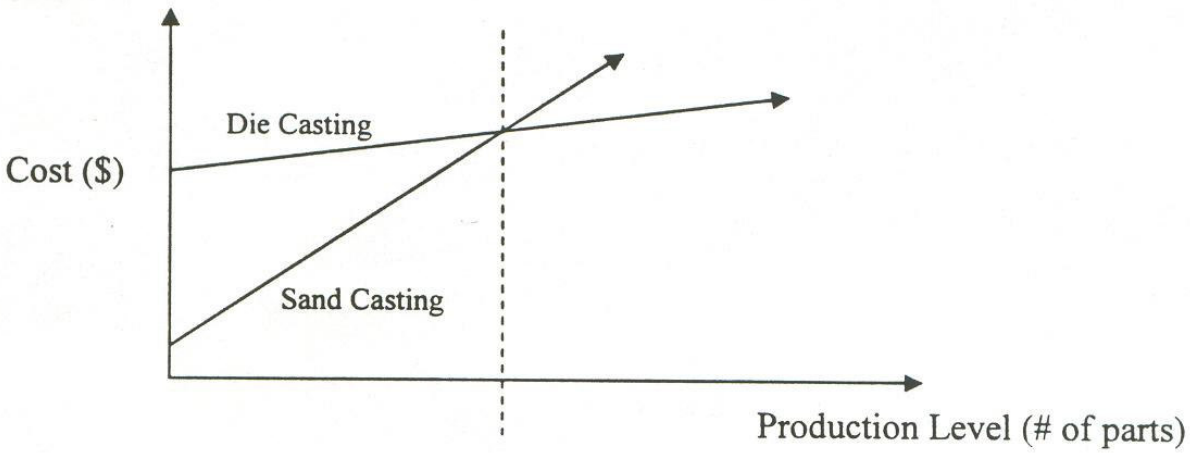
- Equipment: $90,000
- Mold: $30,000 one time
- Foundry metal: $3
- Production Rate: 10 parts per minute

Sand Casting: fixed costs: $25,000 variable costs: $8

(Total cost C_T) = $25,000 + $8*(Production Level or Q)

Die Casting: fixed costs: $120,000 variable costs: $3

C_T = $120,000 + $3*(Q)



Break Even Point

The break even point is the production level where the total costs of the two options are equal:

$$25{,}000 + 8*Q = 120{,}000 + 3*Q$$

or

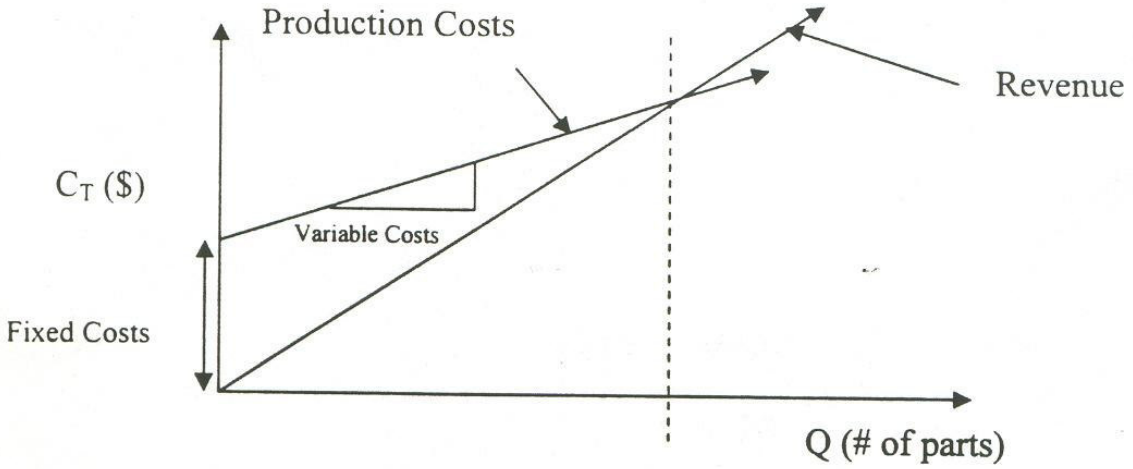
$$Q = 19{,}000$$

How long will it take to reach break even for each production system (assuming the plant runs 8 hr per day 6 days per week?

Sand Casting: 19,000/10/8/6 = 39.6 weeks

Die Casting: 19,000/10/60/8 = 4 days

Note that these two production systems are very different economically. They are also very different physically. Many sand cast parts cannot be die cast and vice versa. The business organization for the two systems would probably be different (Die casting machines often run 2 or 3 shifts per day).



Break Even Point

Rate of Return:

Simply write the present value equations for a problem but leave the interest rate as the variable "i". Solve for "i" to find the actual interest rate corresponding to a given cash flow. This interest rate is the Rate of Return.

Simple Payback:

Ignoring interest, calculate the time required for income from an investment to pay back the initial investment.