Midterm 1
ME EN 3210 – Spring 2004
Monday, February 23, 2002

Notes:
1. **DO NOT OPEN THIS EXAM UNTIL YOU ARE NOTIFIED.**
2. Permitted resources: one half page (8.5" x 5.5" ≈ 46.76 sq. in. single sided) of notes, calculator, pencil, and eraser. This is a closed book exam.
3. A 50 minute period will be provided to take this exam.
4. Read each question **CAREFULLY and COMPLETELY** before starting.

Name: _______________________

Student Number: _______________________

\[
\begin{array}{c}
\text{#1: } 24.3 \\
\text{#2: } 23.7 \\
\text{#3: } 28.6 \\
\text{TOT: } 86.6
\end{array}
\]
1) A mechanism from a nail gun is shown below. The system is driven by the velocity source, \( V_1 \) applied to the drive shaft. The shaft slides through a sliding bearing, \( B_1 \), in the lever arm and a spring, \( K \), compliently couples translation of the shaft and the end of the lever arm. The lever arm pivots on a rotational bearing with damping \( B_2 \). The hammer, represented by the mass, \( M \), is fixed to the opposite end of the arm.

![Mechanism Diagram]

a) Place nodes for your linear graph in the space below. Clearly indicate all velocities on the figure above and the linear graph. (10 pts)

b) Add elements to the linear graph modeling all of the features of the system described above. Label all elements on the linear graph and list governing equations for any transformer or gyrator elements (20 points).

\[
\omega_{12} = \frac{V_I}{J_2} \\
T_{12} = -J_2 F_2 \\
F_{14} = \frac{1}{J_1} T_{13} \\
U_{11} = -J_1 \omega_{13}
\]
2) A simple shock absorber is shown in the diagram and modeled by the linear graph below. It consists of a hydraulic cylinder and piston with an orifice in the piston that permits fluid to flow between the two sides of the cylinder as the velocities, \( v_2 \) and \( v_3 \), and pressures, \( P_2 \) and \( P_1 \), vary. Assume that the piston has area, \( A \), and that the orifice provides fluid resistance, \( R \). The piston/cylinder is described by typical gyrator equations (\( Q_R = v_x A \) and \( R = \frac{1}{A} F_R \)) with the subtlety that the fluid flow is now attributed to flow through the orifice. Determine the translational damping coefficient of the shock absorber. (i.e.: find \( B \) for a typical mechanical damper where \( F = Bv \)). (30 pts)

![Diagram of a shock absorber model](image)

\[
\begin{align*}
\text{From the graph:} & \quad Q_Y = P_0 = \frac{1}{A} F_A = -\frac{1}{A} F \\
\text{KVL:} & \quad P_0 = P_R \\
\text{Subs for} \ P_2 = Q_0 \cdot R = -\frac{1}{A} F \\
\text{KCL@} \ P_2: & \quad Q_B = -Q_R \\
\text{Subs for} \ Q_0: & \quad R \ A = \frac{1}{A} F
\end{align*}
\]

\[
F = R \ A^2 \ (v_2 - v_1)
\]

\[\text{Equivalent:} \ B = -2\]
3) A diagram of a geared DC motor and load is shown below with a simplified linear graph.

![Diagram of a DC Motor and Load]

a) Based on the linear graph, derive the transfer function \( \frac{e_2(\omega)}{i_r(\omega)} \) (30 points).

\[
\begin{align*}
\text{ELEM:} & \quad T_J = J_5 \omega_d \quad T_K = K \frac{1}{\beta} \omega_e \\
& \quad T_E = B \omega_d \\
& \quad \omega_e = -N \omega_d \quad T_C = \frac{1}{N} T_d \\
& \quad T_B = -K_T \alpha \\
& \quad \omega_B = \frac{K_T}{K_T} \omega_a
\end{align*}
\]

KCL @ \( \omega_B \):

\[ T_K - T_J - T_E = 0 \Rightarrow \frac{1}{K} T_K - J_5 \omega_d - B \omega_d = 0 \quad (1) \]

From the elements, where \( \omega_B = \omega_d = \omega_c = \omega_a = \omega_d \):

\[ (2) \]

Also by KVL (1):

\[ W_d = W_a = W_c = W_0 = 0 \]

\[ (2) \]

KVL (2):

\[ V_5 - V_a = 0 \Rightarrow V_5 = V_a = K_T W_d \]

\[ (3) \]

KVL (3):

\[ V_5 - V_a = 0 \Rightarrow V_5 = V_a = K_T W_d \]

\[ \Rightarrow W_B = \frac{V_5}{K_T} \quad (3) \]

b) The above model is overly simplified. If you could add one element to the model to improve its accuracy the most, what would it be and why? (one sentence justification) (10 points)

- Motor resistance, \( R \)
- While other terms affect dynamics slightly, \( R \) directly influences the steady state speed, accuracy.

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