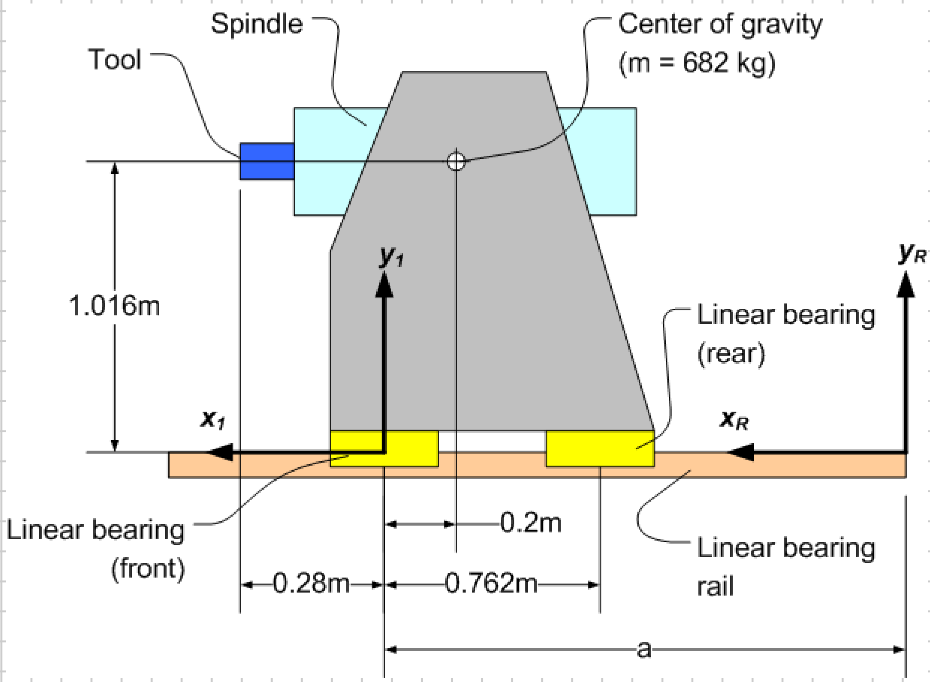
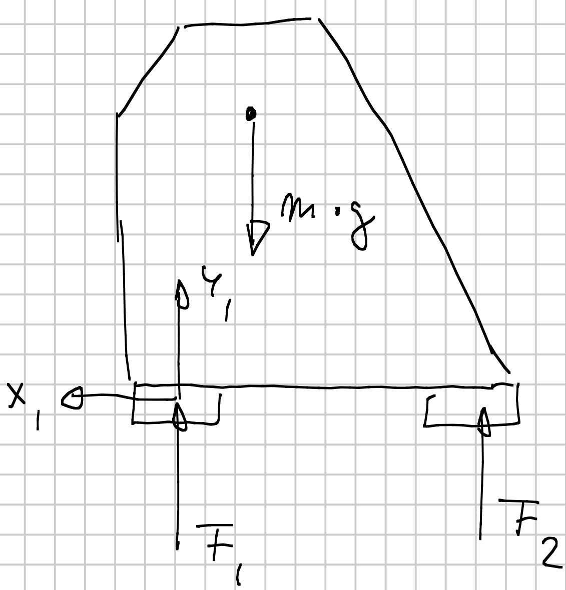
Problem Set 1 – Inertial error of single axis

Note Title 9/5/2006



a) find forces on bearings :

FBD :



$$\sum F_y \overset{!}{=} 0 : \quad F_1 + F_2 - m \cdot g = 0 \qquad (1)$$

$$\sum M_z \stackrel{!}{=} 0 : \quad -m \cdot g \cdot 0.2\,m + F_2 \cdot 0.762\,m = 0$$

$$\rightarrow F_2 = \frac{m \cdot g \cdot 0.2\,m}{0.762\,m} = \frac{682\,kg \cdot 9.81\,\frac{m}{s^2} \cdot 0.2\,m}{0.762\,m}$$

$$\boxed{F_2 = 1756.0\,N} \quad (2)$$

(2) in (1) :

$$F_1 = m \cdot g - F_2 = 682\,kg \cdot 9.81\,\frac{m}{s^2} - 1756\,N$$

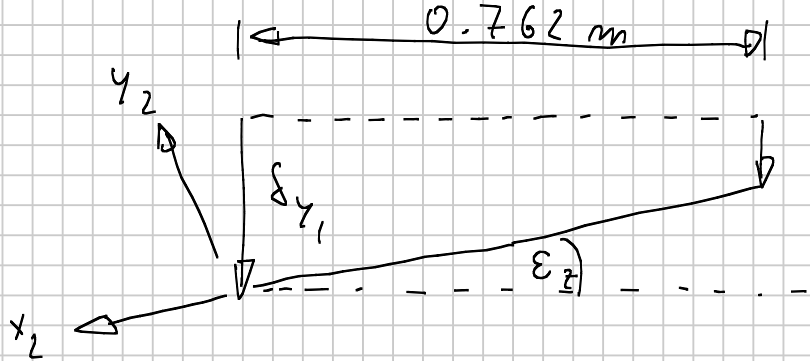
$$\boxed{F_1 = 4934.4\,N}$$

The weight of the axis puts the linear bearings into compression. The resulting displacement is:

$$\delta_v = -\frac{F_v}{k_v} \quad \rightarrow \quad \delta_{y_1} = -\frac{4934.4\,N}{350 \cdot 10^6\,\frac{N}{m}} = -1.41 \cdot 10^{-5}\,m$$

$$\delta_{y_2} = -\frac{1756.0\,N}{350 \cdot 10^6\,\frac{N}{m}} = -5.02 \cdot 10^{-6}\,m$$

Find rotation as a result of bearing displacement.



$$\varepsilon_z = \frac{\delta_{y_1} - \delta_{y_2}}{0.762\,m} = \frac{-1.41 \cdot 10^{-5}\,m + 5.02 \cdot 10^{-6}\,m}{0.762\,m} = \underline{\underline{-1.19 \cdot 10^{-5}\,rad}}$$

HTM of linear axis: $\begin{bmatrix} 1 & -\varepsilon_z & \varepsilon_y & a + \delta_x \\ \varepsilon_z & 1 & -\varepsilon_x & b + \delta_y \\ -\varepsilon_y & \varepsilon_x & 1 & c + \delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$

here: $\varepsilon_x = \varepsilon_y = 0$, $\varepsilon_z = -1.19 \cdot 10^{-5}\,rad$

$\delta_x = \delta_z = 0$, $\delta_y = \delta_{y_1} = -1.41 \cdot 10^{-5}\,m$

$$\Rightarrow R_{1R} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ HTM from reference to axis

$$R_{21} = \begin{bmatrix} 1 & 1.19 \cdot 10^{-5} & 0 & 0 \\ -1.19 \cdot 10^{-5} & 1 & 0 & -1.41 \cdot 10^{-5} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ HTM as a function of bearing displacement

$$\Rightarrow R_{2R} = R_{1R} \cdot R_{21} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1.19 \cdot 10^{-5} & 0 & 0 \\ -1.19 \cdot 10^{-5} & 1 & 0 & -1.41 \cdot 10^{-5} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_{2R} = \begin{bmatrix} 1 & 1.19\cdot 10^{-5} & 0 & a \\ -1.19\cdot 10^{-5} & 1 & 0 & -1.41\cdot 10^{-5} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Position of tool tip with error:

$$P_{tip,R,error} = R_{2R} \cdot P_{tip} = \begin{bmatrix} 1 & 1.19\cdot 10^{-5} & 0 & a \\ -1.19\cdot 10^{-5} & 1 & 0 & -1.41\cdot 10^{-5} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.28 \\ 1.016 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} a + 0.280012 \\ 1.015983 \\ 0 \\ 1 \end{bmatrix} \rightarrow P_{tip,R,error} = \begin{pmatrix} a + 0.280012 \\ 1.015983 \\ 0 \end{pmatrix}$$

Position of tool tip without error:

$$P_{tip,R,noerror} = R_{1R} \cdot P_{tip} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.28 \\ 1.016 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a + 0.28 \\ 1.016 \\ 0 \\ 1 \end{bmatrix}$$

$$\rightarrow P_{tip,R,no\ error} = \begin{pmatrix} a + 0.28 \\ 1.016 \\ 0 \end{pmatrix}$$

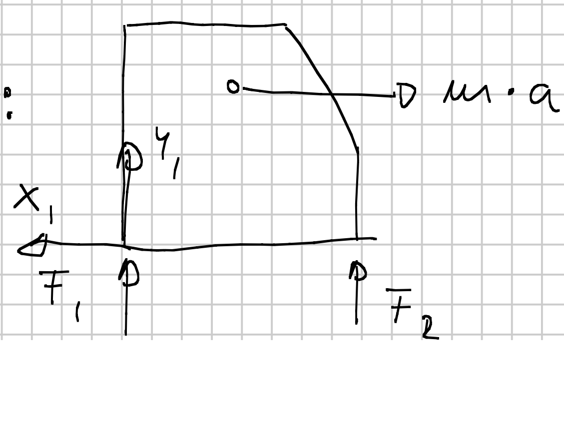
The error at the tooltip is found as the difference between $P_{tip,R,error}$ and $P_{tip,R,no\ error}$

$$\Delta P = P_{tip,R,error} - P_{tip,R,no\ error} = \begin{pmatrix} a + 0.280012 \\ 1.015983 \\ 0 \end{pmatrix} - \begin{pmatrix} a + 0.28 \\ 1.016 \\ 0 \end{pmatrix}$$

$$\boxed{\rightarrow \Delta P = \begin{pmatrix} 1.21 \cdot 10^{-5} \\ -1.74 \cdot 10^{-5} \\ 0 \end{pmatrix}}$$

b) after calibration, the effects of gravity are elimin.

$\rightarrow$ FBD:



$$\sum F_y \overset{!}{=} 0: \quad F_1 + F_2 = 0 \quad (1)$$

$$\sum M_z \overset{!}{=} 0: \quad -m \cdot a \cdot 1.016\,m + F_2 \cdot 0.762\,m = 0$$

$$\Rightarrow F_2 = \frac{m \cdot a \cdot 1.016\,m}{0.762\,m} = \frac{682\,kg \cdot 0.5 \cdot 9.81\,\frac{m}{s^2} \cdot 1.016\,m}{0.762\,m}$$

$$\boxed{F_2 = 4460.3\,N} \quad (2)$$

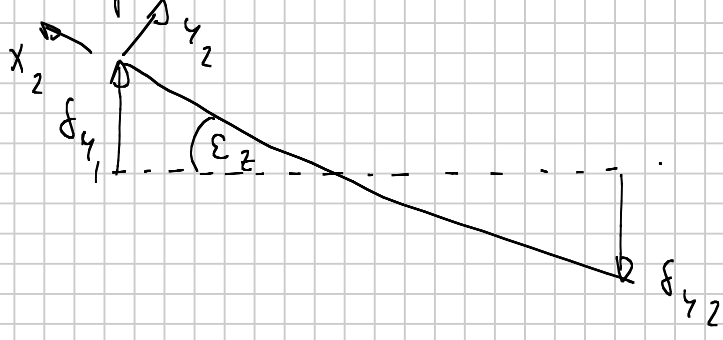
(2) in (1): $\boxed{F_1 = -4460.3\,N}$

Find bearing displacements:

$$\delta_{y_1} = -\frac{F_1}{k_1} = \frac{4460.3\,N}{350 \cdot 10^6\,N/m} = 1.27 \cdot 10^{-5}\,m$$

$$\delta_{y_2} = -\frac{F_2}{k_1} = \frac{-4460.3\,N}{350 \cdot 10^6\,N/m} = -1.27 \cdot 10^{-5}\,m$$

Find rotation of axis:



$$\varepsilon_2 = \frac{\delta_{y_1} - \delta_{y_2}}{0.762} = \frac{1.27 \cdot 10^{-5} + 1.27 \cdot 10^{-5}}{0.762} = \underline{\underline{3.34 \cdot 10^{-5} \text{ rad}}}$$

$$\rightarrow R_{21} = \begin{bmatrix} 1 & -3.34 \cdot 10^{-5} & 0 & 0 \\ 3.34 \cdot 10^{-5} & 1 & 0 & 1.27 \cdot 10^{-5} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

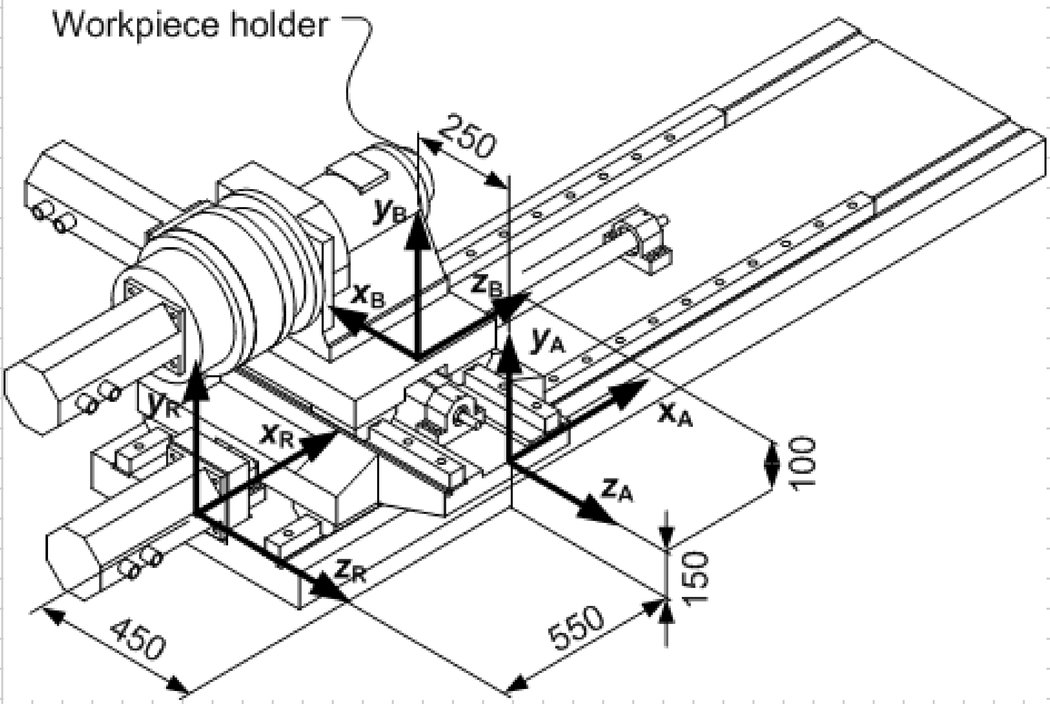
$$\rightarrow P_{tip,R,error} = R_{1R} \cdot R_{21} \cdot P_{tip,2} = \begin{bmatrix} 1 & -3.34 \cdot 10^{-5} & 0 & a \\ 3.34 \cdot 10^{-5} & 1 & 0 & 1.27 \cdot 10^{-5} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.28 \\ 1.016 \\ 0 \\ 1 \end{bmatrix}$$

$$P_{tip,R,error} = \begin{pmatrix} a + 0.279966 \\ 1.016022 \\ 0 \end{pmatrix}$$

$$\boxed{\rightarrow \Delta P = \begin{pmatrix} -3.39 \cdot 10^{-5} \text{ m} \\ 2.21 \cdot 10^{-5} \text{ m} \\ 0 \end{pmatrix}}$$

Problem Set 1 - Two stacked axes

Note Title 9/5/2006

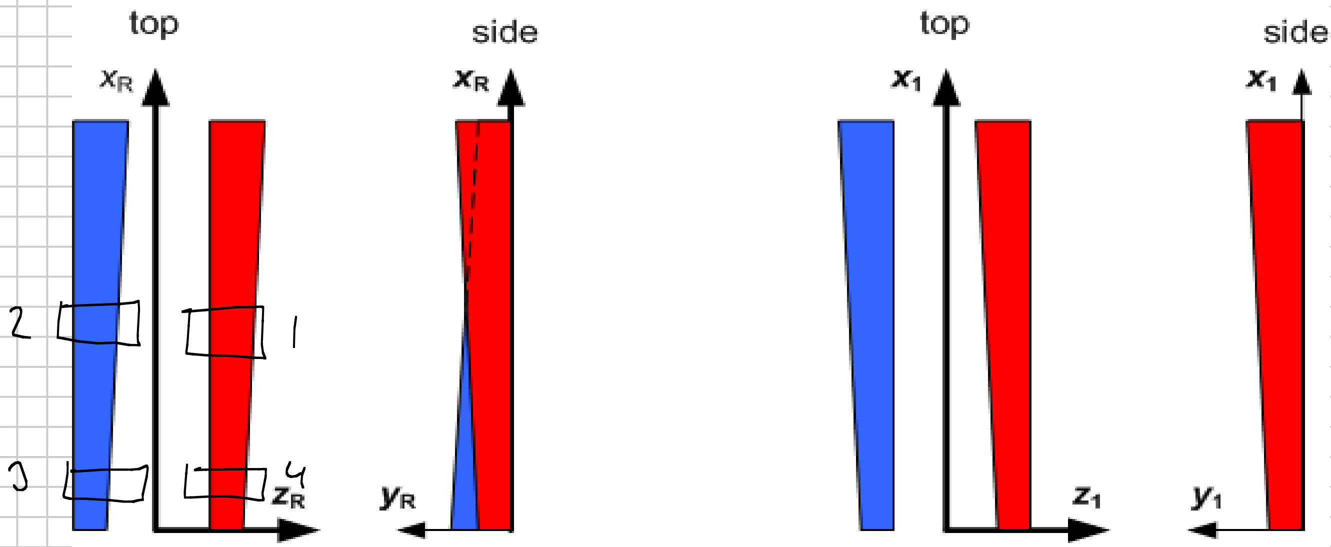


a) HTM from reference to axis A:

$$R_{AR} = \begin{bmatrix} 1 & 0 & 0 & a+0.55 \\ 0 & 1 & 0 & 0.15 \\ 0 & 0 & 1 & 0.45 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

HTM from axis A to axis B

$$R_{BA} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0.1 \\ -1 & 0 & 0 & -b-0.25 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Bottom Axis (x-axis)

Top Axis (y-axis)

Axis A:

$$\delta_{AY} = \frac{\delta_{AY1} + \delta_{AY2} + \delta_{AY3} + \delta_{AY4}}{4} = \frac{\delta_A - \delta_A - \delta_A + \delta_A}{4} = 0$$

$$\delta_{AZ} = \frac{\delta_{AZ1} + \delta_{AZ2} + \delta_{AZ3} + \delta_{AZ4}}{4} = \frac{\delta_A + \delta_A + \delta_A + \delta_A}{4} = \delta_A$$

$$\varepsilon_{Ax} = \frac{\frac{\delta_{AY2} + \delta_{AY3}}{2} - \frac{\delta_{AY1} + \delta_{AY4}}{2}}{W_A} = \frac{\frac{-\delta_A - \delta_A}{2} - \frac{\delta_A + \delta_A}{2}}{0.35} = \frac{-2\delta_A}{0.35}$$

$$\varepsilon_{AY} = \frac{\frac{\delta_{AZ3} + \delta_{AZ4}}{2} - \frac{\delta_{AZ1} + \delta_{AZ2}}{2}}{L_A} = \frac{\frac{\delta_A + \delta_A}{2} - \frac{\delta_A + \delta_A}{2}}{0.45} = 0$$

$$\varepsilon_{AZ} = \frac{\frac{\delta_{AY1} + \delta_{AY2}}{2} - \frac{\delta_{AY3} + \delta_{AY4}}{2}}{L} = \frac{\frac{\delta_A - \delta_A}{2} - \frac{-\delta_A + \delta_A}{2}}{0.45} = 0$$

$$\rightarrow R_{AA'} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{2\delta_A}{0.35} & 0 \\ 0 & \frac{-2\delta_A}{0.35} & 1 & \delta_A \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

B-Axis:

$$\delta_{By} = \frac{\delta_{By_1} + \delta_{By_2} + \delta_{By_3} + \delta_{By_4}}{4} = \frac{\delta_B + \delta_B + \delta_B + \delta_B}{4} = \delta_B$$

$$\delta_{Bz} = \frac{\delta_{Bz_1} + \delta_{Bz_2} + \delta_{Bz_3} + \delta_{Bz_4}}{4} = \frac{-\delta_B - \delta_B - \delta_B - \delta_B}{4} = -\delta_B$$

$$\varepsilon_{Bx} = \frac{\frac{\delta_{By_2} + \delta_{By_3}}{2} - \frac{\delta_{By_1} + \delta_{By_4}}{2}}{W_B} = \frac{\frac{+\delta_B + \delta_B}{2} - \frac{+\delta_B + \delta_B}{2}}{0.25} = 0$$

$$\varepsilon_{By} = \frac{\frac{\delta_{Bz_3} + \delta_{Bz_4}}{2} - \frac{\delta_{Bz_1} + \delta_{Bz_2}}{2}}{L_B} = \frac{\frac{-\delta_B - \delta_B}{2} - \frac{\delta_B - \delta_B}{2}}{0.3} = 0$$

$$\varepsilon_{Bz} = \frac{\frac{\delta_{By_1} + \delta_{By_2}}{2} - \frac{\delta_{By_3} + \delta_{By_4}}{2}}{L_B} = \frac{\frac{+\delta_B + \delta_B}{2} - \frac{\delta_B + \delta_B}{2}}{0.3} = 0$$

$$\rightarrow R_{BB'} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & +\delta_B \\ 0 & 0 & 1 & -\delta_B \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

To find error at the work piece:

$$P_{WP,R,\text{no error}} = R_{AR} \cdot R_{BA} \cdot P_{WP,B}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1.8 \\ 0 & 1 & 0 & 0.45 \\ 0 & 0 & 1 & 0.15 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0.1 \\ -1 & 0 & 0 & -0.6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.115 \\ 0.325 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.125 \\ 0.365 \\ -0.35 \\ 1 \end{bmatrix}$$

$$P_{WP,R,\text{error}} = R_{AR} \cdot R_{AA'} \cdot R_{BA} \cdot R_{BB'} \cdot P_{WP,B}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1.8 \\ 0 & 1 & 0 & 0.45 \\ 0 & 0 & 1 & 0.15 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{2\delta_A}{0.35} & 0 \\ 0 & -\frac{2\delta_A}{0.35} & 1 & \delta_A \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0.1 \\ -1 & 0 & 0 & -0.6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & +\delta_B \\ 0 & 0 & 1 & -\delta_B \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.115 \\ 0.325 \\ 1 \end{bmatrix}$$

normal grade: $\Delta P = \begin{pmatrix} -1.75 \cdot 10^{-5} \\ -1.01 \cdot 10^{-4} \\ -5.94 \cdot 10^{-6} \end{pmatrix}$ $\quad |\Delta P| = 1.03 \cdot 10^{-4}\ m$

high grade: $\Delta P = \begin{pmatrix} -1.2 \cdot 10^{-5} \\ -7.03 \cdot 10^{-5} \\ -4.11 \cdot 10^{-6} \end{pmatrix}$ $\quad |\Delta P| = 7.1 \cdot 10^{-5}\ m$

precision grade: $\Delta P = \begin{pmatrix} -5.5 \cdot 10^{-6} \\ -4.71 \cdot 10^{-5} \\ -2.63 \cdot 10^{-6} \end{pmatrix}$ $|\Delta P| = 4.75 \cdot 10^{-5}\ m$

super precision grade: $\Delta p = \begin{pmatrix} -2.5 \cdot 10^{-6} \\ -3.18 \cdot 10^{-5} \\ -1.7 \cdot 10^{-6} \end{pmatrix}$ $|\Delta P| = 3.19 \cdot 10^{-5}\ m$

ultra precision grade: $\Delta p = \begin{pmatrix} -10 \cdot 10^{-7} \\ -1.04 \cdot 10^{-5} \\ -5.7 \cdot 10^{-7} \end{pmatrix}$ $|\Delta p| = 1.05 \cdot 10^{-5}\ m$

Problem Set 1 – Where to place the precision and normal

Note Title 9/25/2006

Because of the amplification of rotational errors due to the height above the bearings, the rails that are located the furthest away from the point of interest are the most crucial

→ use precision grade for bottom axis
use normal grade for top axis

Proof: Using the HTM from Problem 2:

$$\Delta P_1 = \begin{pmatrix} 1.75 \cdot 10^{-5} \\ -3.53 \cdot 10^{-5} \\ -2.63 \cdot 10^{-5} \end{pmatrix} = \begin{pmatrix} 17.5\ \mu m \\ -35.3\ \mu m \\ -2.63\ \mu m \end{pmatrix} \mathrel{\hat{=}} \underline{\underline{39.3\ \mu m}}$$

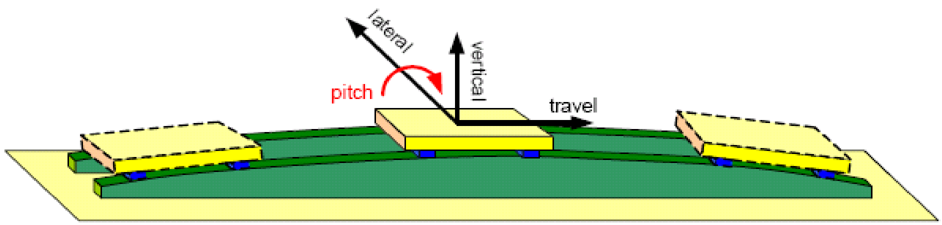
(precision at bottom normal at top)

$$\Delta P_2 = \begin{pmatrix} -5.5 \cdot 10^{-5} \\ -1.134 \cdot 10^{-4} \\ -5.94 \cdot 10^{-6} \end{pmatrix} = \begin{pmatrix} -5.5\ \mu m \\ -113.4\ \mu m \\ -5.94\ \mu m \end{pmatrix} \mathrel{\hat{=}} \underline{\underline{113.6\ \mu m}}$$

(precision at top normal at bottom)

Problem Set 1 – How to set up a HTM for bowed rails

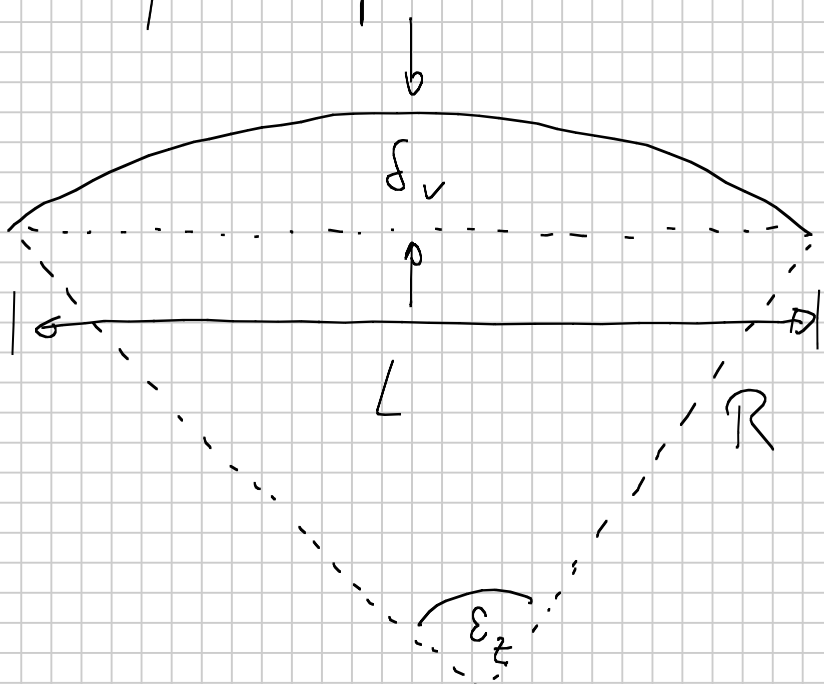
Note Title 9/25/2006



Pitch as a result of a bow of the rails in the vertical direction

For bowed rails, as shown above, the method to develop a HTM as used so far will produce the wrong result. That is because the vertical errors at the start and end position are virtually equal, resulting in zero displ. in the vertical axis and zero rotation about the horizontal axis (pitch).

In this case, the pitch needs to be identified differently



$$R = \frac{\delta_v}{2} + \frac{L^2}{8\delta_v}$$

$$L = 2R \sin\frac{\varepsilon_t}{2} \rightarrow \boxed{\varepsilon_t = 2 \cdot \operatorname{asin}\left(\frac{L}{2R}\right)}$$

for precision grade : $\delta_v = 26 \cdot 10^{-6}$ m , $L = 1.25$ m

$$\rightarrow R = \frac{26 \cdot 10^{-6}}{2} + \frac{(1.25)^2}{8 \cdot 26 \cdot 10^{-6}} = 7'512.019244 \text{ m}$$

$$\rightarrow \boxed{\varepsilon_t = 9.534 \cdot 10^{-3} \text{ rad}}$$