Fundamentals of Errors

ME EN 7960 – Precision Machine Design
Topic 2

Accuracy, Repeatability, and Resolution

Resolution

Accuracy

Repeatability

Target Point

Positions achieved while attempting to position to target point plus finest programmable increment of motion

Positions achieved while attempting to position to target point
Accuracy

• Accuracy is the ability to tell the truth or:
   – The maximum translational or rotational error between any two points in the machine's work volume
   – Linear, planar, and volumetric accuracy can all be defined for a machine
• Note: mechanical accuracy is far more costly than repeatability

Repeatability

• Repeatability (precision) is the ability to tell the same story over and over again or:
   – The error between a number of successive attempts to move the machine to the same position
   – Repeatability is often considered to be the most important parameter of a computer controlled machine (or sensor)
   – Often the intent is to map the errors and then compensate for them
• Minimize static friction, backlash, and thermal variants to get better repeatability
Resolution

- Resolution is how detailed your story is
  - Resolution is the larger of the smallest programmable step or:
  - The smallest mechanical step the machine can make during point to point motion
  - Resolution gives a lower bound on the repeatability
- Minimize static friction and use higher resolution feedback to get better resolution

What is the importance of errors?

- The trajectory of the tool relative to the work piece directly determines the dimensions of all features machined
- No machine tool monitors the tool trajectory directly. Instead, the tool trajectory is monitored indirectly by monitoring the position of the moving components (axes)
- Any deviation of any moving component will imprinted onto the work piece, thereby reducing the part accuracy
Case 1: Perfectly straight z-axis bearings create a perfectly cylindrical part.

Case 2: The straightness errors of the z-axis bearings are transferred onto the part.
Performance Limiting Criteria

• Accuracy and Repeatability are limited by:
  – Geometric errors of all components
  – Kinematic errors
  – Load induced errors
  – Thermal errors
  – Dynamic errors
  – Calibration errors
  – Computational errors

• Resolution is limited by
  – Quality of sensors
  – Quality of control system
  – Friction (stick and slip effect)
  – Backlash

Geometric Errors

• Errors in the form of individual machine components:
  – Component straightness error due to machining errors
  – Component straightness error due to gravity loading

• Surface finish effects

• Quasi-static accuracy of surfaces moving relative to each other (e.g., linear or rotary motion axes):
  – Linear motion axis:
    • Pitch
    • Roll
    • Yaw
    • Straightness (2 components)
    • Linear displacement
Geometric Error - Gravity

Geometric Errors - Roll

Roll as a result of different slopes of the rails
Geometric Errors - Pitch

Pitch as a result of a bow of the rails in the vertical direction

Geometric Errors - Yaw

Yaw as a result of bow of the rails in the lateral direction
Other Errors

- Kinematic Errors
  - Errors in an axis’s trajectory that are caused by misaligned or improperly sized components
    - Orthogonality between axes
    - Parallelism between axes
    - Error motions in a closed kinematic chain

- External load induced errors:
  - Errors due to deformation of components:
    - Gravity load induced errors
    - Cutting/probing force induced errors
    - Axis acceleration load induced errors

\[
\Delta z = b \cdot \gamma
\]
Geometric Error – Load Induced

Thermal Errors

- Mean temperature other than 68 °F (20 °C)
  - Gradients in environment's temperature
  - Errors caused by thermal expansion of elements:
    - External heat sources:
      - Mean temperature of the room
      - Sun shining through the window onto the machine
      - Nearby machine's hot air vent
      - Overhead lights
      - Operator's body heat
    - Internal heat sources:
      - Motors
      - Bearings
      - Machining process
      - Pumps
      - Expansion of compressed fluids
      - Coolant
Dynamic Errors

- Errors caused by vibration or control processes:
  - Vibration:
    - External environment (usually through the ground)
    - Cutting process
    - Rotating masses
  - Control system:
    - Algorithm type (e.g., PID, adaptive, etc.)
    - Stick-slip friction
    - Varying mass
    - Varying stiffness
  - Switching amplifiers
    - Servo loop frequency excites a natural mode of the machine
Calibration Errors

- Errors associated with sensors:
  - Intrinsic accuracy
  - Interpolation
  - Mounting errors:
    - Position
    - Mounting stress
  - Calibration (error associated with the mastering process)

Additional Errors

- Computational errors:
  - Error introduced in the analysis algorithms
  - Rounding off errors due to hardware
- Additional sources of error (often very difficult to model):
  - Humidity
  - Loose joints
  - Dirt
- Variations in supply systems:
  - Electricity
  - Fluid pressure
  - Operator inattention
  - Fluid supply cleanliness
- Operators (“52-2” factor)
Error Assessment and Budgeting

- Given all the different types of errors that can affect all different components:
  - Keeping track of all the errors is such a daunting task:
    - Most engineers don’t bother and use “experience” to guide the design
    - It is left up to manufacturing and service to work the bugs out
  - This seems to be a major source of reliability and performance problems
- The solution to a successful project is a good budget:
  - A project requires a good financial budget to make it feasible
  - A project requires a good time budget to make it feasible
  - A project requires a good error budget to make it feasible
- In order to make a good error budget for the system, a good mathematical model is needed

Homogenous Transformation Matrices

- Allows the designer to consider one part of the machine at a time, and then link them all together like beads on a string
- Based on rigid body model of a linear series (open chain) of coordinate frames
- Takes into account linear and angular offsets between coordinate frames
- Transforms XYZ coordinates of one frame into XYZ coordinates of another frame
- Used to transform locally referenced errors into errors referenced with respect to the tool point and the work piece
Homogenous Transformation Matrices (contd.)

- Coordinate frames are placed at bearings, joints, and areas where other parameters are lumped.
- Closed chains (e.g., a five point bearing mount) need to be modeled with the generation of constraint equations.

Structure of HTM’s

\[
R_{T_n} = \begin{bmatrix}
O_{n1} & O_{n2} & O_{n3} & P_x \\
O_{j1} & O_{j2} & O_{j3} & P_y \\
O_{k1} & O_{k2} & O_{k3} & P_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

- Orientation of \(Y_n\) with respect to adjacent coordinate frame.
- Orientation of \(X_n\) with respect to adjacent coordinate frame.
- Orientation of \(Z_n\) with respect to adjacent coordinate frame.
- Translation of \(X_nY_nZ_n\) with respect to adjacent coordinate frame.
Structure of HTM’s (contd.)

- The first three columns are the direction cosines (unit vectors i, j, k).
  - They represent the orientation of the $X_n$, $Y_n$, and $Z_n$ axes with respect to an adjacent coordinate frame.
- The last column is the position of the rigid body's coordinate system's origin with respect to the reference frame.
- The pre-superscript represents the reference frame in which you want the result to be represented.
- The post-subscript represents the reference frame from which you are transferring.

Structure of HTM’s (contd.)

- The equivalent coordinates of a point in a reference frame $n$, in a reference frame $R$ are:

$$
\begin{bmatrix}
X_R \\
Y_R \\
Z_R \\
1
\end{bmatrix} = R_{T_n} \begin{bmatrix}
X_n \\
Y_n \\
Z_n \\
1
\end{bmatrix}
$$

HTM that describes the transformation of reference frame $n$ into reference frame $R$. 
Example: Translation in X

• The coordinate system $X_YZ$ is shifted in $X$ by an amount $a$:

\[
\begin{bmatrix}
1 & 0 & 0 & a \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
Example: Rotation about X-Axis

- the $X_1Y_1Z_1$ coordinate system is rotated by an amount $\theta_x$ about the $X$ axis:

\[
\begin{bmatrix}
X_R \\
Y_R \\
Z_R
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \theta_x & -\sin \theta_x & 0 \\
0 & \sin \theta_x & \cos \theta_x & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
X_1 \\
Y_1 \\
Z_1
\end{bmatrix}
\]
Sequential Systems

- Transformation from the Nth axis to the reference system will be the sequential product of all the HTMs:

\[ R_{TN} = \prod_{m=1}^{N} m^{-1} T_m = T_1 T_2 T_3 \ldots \]

- The error between the tool and the work piece is obtained from:

\[
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z \\
\end{bmatrix} = \begin{bmatrix}
P_x \\
P_y \\
P_z \\
\end{bmatrix}_{work,R} - \begin{bmatrix}
P_x \\
P_y \\
P_z \\
\end{bmatrix}_{tool,R}
\]

Rigid body models of machine components

Source: Alexander Slocum, Precision Machine Design
HTM of Linear Motion Carriage

\[
R_{T_{nerr}} = \begin{bmatrix}
1 & -\varepsilon_z & \varepsilon_y & a + \delta_x \\
\varepsilon_z & 1 & -\varepsilon_x & b + \delta_y \\
-\varepsilon_y & \varepsilon_x & 1 & c + \delta_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Estimating Position Errors from Catalogs

Source: Alexander Stocum, Precision Machine Design
The translational errors are based on the average of the straightness errors experienced by the bearing block:

\[
\begin{aligned}
\delta_{\text{axial}} &= \delta_{\text{axial}} \\
\delta_{\text{lateral}} &= \frac{\delta_{\text{lateral1}} + \delta_{\text{lateral2}} + \delta_{\text{lateral3}} + \delta_{\text{lateral4}}}{4} \\
\delta_{\text{vertical}} &= \frac{\delta_{\text{vertical1}} + \delta_{\text{vertical2}} + \delta_{\text{vertical3}} + \delta_{\text{vertical4}}}{4}
\end{aligned}
\]
The angular errors are based on the differences in the average straightness errors experienced by pairs of bearing blocks acting across the carriage:

\[
\varepsilon_{\text{axial}} = \frac{\frac{\delta_{\text{vertical}2} + \delta_{\text{vertical}3} - \delta_{\text{vertical}1} + \delta_{\text{vertical}4}}{2}}{W}
\]

\[
\varepsilon_{\text{lateral}} = \frac{\frac{\delta_{\text{vertical}1} + \delta_{\text{vertical}2} - \delta_{\text{vertical}3} + \delta_{\text{vertical}4}}{2}}{L}
\]

\[
\varepsilon_{\text{vertical}} = \frac{\frac{\delta_{\text{vertical}1} + \delta_{\text{vertical}4} - \delta_{\text{vertical}3} + \delta_{\text{vertical}2}}{2}}{L}
\]

Note we assumed for the straightness we assumed all the errors are acting in the same direction. Here we assume one set of errors acts up, and the other acts down. This is very conservative (makes up for other effects we might miss).
Linear Error Plot

Straightness error of a kinematically (statically determined) supported linear axis (5 rolling element bearings on a vee and flat)

Fourier Transformation of Linear Error Plot

- The Fourier Transformations when plotted as the error amplitude as a function of wavelength is an invaluable diagnostic tool

- The wavelength of rolling elements is $2\pi D$
HTM of Rotary Axis

\[ R_{\text{err}} = \begin{bmatrix} \cos \varepsilon_x \cos \theta_z & -\cos \varepsilon_x \sin \theta_z & \sin \varepsilon_x & \delta_z \\ \sin \varepsilon_x \sin \theta_z + \cos \varepsilon_x \sin \theta_z & \cos \varepsilon_x \cos \theta_z - \sin \varepsilon_x \sin \theta_z & -\sin \varepsilon_x & \delta_y \\ -\cos \varepsilon_x \sin \theta_z + \sin \varepsilon_x \sin \theta_z & \sin \varepsilon_x \cos \theta_z + \cos \varepsilon_x \sin \theta_z & \cos \varepsilon_x & \delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

Source: Alexander Slocum, Precision Machine Design

Rotary Errors

- This general result may also be used for the case of a linear motion carriage if \( \varepsilon_z \) is substituted for \( \theta_z \)
- Radial error motion is conservatively estimated to be the ABEC bore tolerance
- Tilt error motion is conservatively estimated to be the ABEC axial tolerance divided by the bearing OD
Rotary Error Plot

- The average error motion is indicative of the form error that will be imparted to the part when held in a lathe spindle.
- The asynchronous error motion is indicative of the surface finish that will be obtained.

Fourier Transformation of Radial Error

- Spindle speed: 1680 rpm (28 Hz)
- Bearing ID: 75 mm
- Bearing OD: 105 mm
- Number of balls: 20
- Ball diameter: 10 mm
- Contact angle: 15
## Combinational Ruler for 3 Common Types

- **Random** - under apparently equal conditions at a given position, errors that do not always have the same value, and can only be expressed statistically.

- **Systematic** - which always have the same value and sign at a given position and under given circumstances.
  - Generally can be correlated with position along an axis and can be corrected.
  - If the relative accompanying random error is small enough.

- **Hysteresis** - a systematic error which in this instance is separated out for convenience.
  - Usually repeatable, sign depends on the direction of approach, and magnitude partly dependent on the travel.
  - May be compensated for if the direction of approach is known and an adequate pre-travel is made.