Modeling Bearings for Selection and Analysis

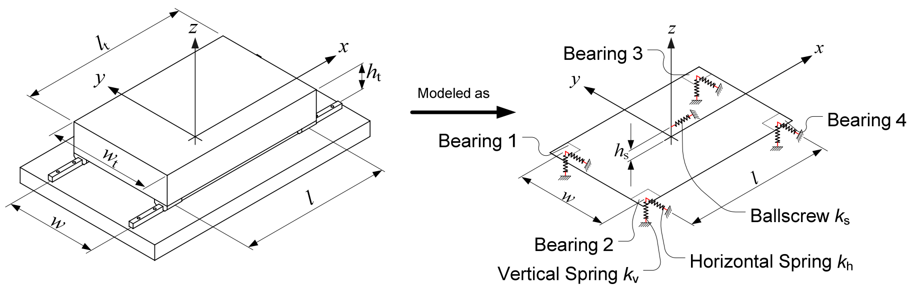
ME EN 7960 – Precision Machine Design
Topic 9

Modeling Principle

- Bearings are complicated machine elements with a large number of rolling elements.
- Not every rolling element makes contact due to manufacturing tolerances.
- Hertzian contact is too difficult and time consuming to model.
- Bearings modeled as steel blocks will exhibit vastly different from real bearings.
- Solution:
  - Rigid body model with bearings modeled as linear springs.
  - Finite element model with bearing modeled as linear springs.
  - Finite element model with bearings modeled as solids having a modulus such that its predicted behavior in finite elements matches that of the real bearing.

Rigid Body Model

- Develop equation of motion



Rigid Body Model

Sum of all kinetic energies:

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 + \frac{1}{2}I_{xx}\dot{\theta}_x^2 + \frac{1}{2}I_{yy}\dot{\theta}_y^2 + \frac{1}{2}I_{zz}\dot{\theta}_z^2$$

Sum of all potential energies:

$$V = \frac{1}{2}k_{v1}z_1^2 + \frac{1}{2}k_{v2}z_2^2 + \frac{1}{2}k_{v3}z_3^2 + \frac{1}{2}k_{v4}z_4^2 + \frac{1}{2}k_{h1}y_1^2 + \frac{1}{2}k_{h2}y_2^2 + \frac{1}{2}k_{h3}y_3^2 + \frac{1}{2}k_{h4}y_4^2 + \frac{1}{2}k_s x_s^2$$

Describe vertical and horizontal displacements of each spring in terms of the global coordinate system:

$$y_1 = y - \frac{l}{2}\theta_z \qquad z_1 = z + \frac{w}{2}\theta_x + \frac{l}{2}\theta_y$$

$$y_2 = y - \frac{l}{2}\theta_z \qquad z_2 = z - \frac{w}{2}\theta_x + \frac{l}{2}\theta_y$$

$$y_3 = y + \frac{l}{2}\theta_z \qquad z_3 = z + \frac{w}{2}\theta_x - \frac{l}{2}\theta_y$$

$$y_4 = y + \frac{l}{2}\theta_z \qquad z_1 = z - \frac{w}{2}\theta_x - \frac{l}{2}\theta_y \qquad x_s = x + h_s\theta_y$$

Rigid Body Model

$$V = \frac{1}{2}k_v\left[4z^2 + w^2\theta_x^2 + l^2\theta_y^2\right] + \frac{1}{2}k_h\left[4y^2 + l^2\theta_z^2\right] + \frac{1}{2}k_s\left[x^2 + h_s^2\theta_y^2 + 2h_s x\theta_y\right]$$

Use Lagrange:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i$$

Equation of Motion:

$$m\ddot{x} + k_s x + k_s h_s \theta_y = F_x$$
$$m\ddot{y} + 4k_h y = F_y$$
$$m\ddot{z} + 4k_v z = F_z$$
$$I_{xx}\ddot{\theta}_x + k_v w^2 \theta_x = T_x$$
$$I_{yy}\ddot{\theta}_y + k_v l^2 \theta_y + k_s h_s^2 \theta_y + k_s h_s x = T_y$$
$$I_{zz}\ddot{\theta}_z + k_h l^2 \theta_z = T_z$$

Rigid Body Model

$$\begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{yy} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \ddot{\theta}_x \\ \ddot{\theta}_y \\ \ddot{\theta}_z \end{bmatrix} +$$

$$\begin{bmatrix} k_s & 0 & 0 & 0 & k_s h_s & 0 \\ 0 & 4k_h & 0 & 0 & 0 & 0 \\ 0 & 0 & 4k_v & 0 & 0 & 0 \\ 0 & 0 & 0 & k_v w^2 & 0 & 0 \\ k_s h_s & 0 & 0 & 0 & k_v l^2 + k_s h_s^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & k_h l^2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ F_z \\ T_x \\ T_y \\ T_z \end{bmatrix}$$

Rigid Body Model

Equation of Motion:

$$\vec{M}\ddot{\vec{q}} + \vec{K}\vec{q} = \vec{Q}$$

Mass Matrix *M*

Acceleration Vector

Stiffness Matrix *K*

Displacement Vector

Load Vector *Q*

Stiffness Matrix *K*:

$$\vec{K} = \begin{bmatrix} k_s & 0 & 0 & 0 & k_s h_s & 0 \\ 0 & 4k_h & 0 & 0 & 0 & 0 \\ 0 & 0 & 4k_v & 0 & 0 & 0 \\ 0 & 0 & 0 & k_v w^2 & 0 & 0 \\ k_s h_s & 0 & 0 & 0 & k_v l^2 + k_s h_s^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & k_h l^2 \end{bmatrix}$$

Rigid Body Model

Quasi-static case: $\ddot{\vec{q}} = \vec{0} \quad \rightarrow \quad \vec{K}\vec{q} = \vec{Q} \quad \rightarrow \quad \vec{q} = \vec{K}^{-1}\vec{Q}$

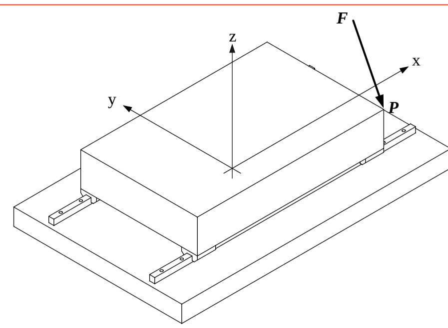
Invert stiffness matrix *K* to obtain compliance matrix *C*:

$$\vec{C} = \begin{bmatrix} \frac{k_v l^2 + k_s h_s^2}{k_v k_s l^2} & 0 & 0 & 0 & \frac{-h_s}{k_v l^2} & 0 \\ 0 & \frac{1}{4k_h} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4k_v} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k_v w^2} & 0 & 0 \\ \frac{-h_s}{k_v l^2} & 0 & 0 & 0 & \frac{1}{k_v l^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{k_h l^2} \end{bmatrix}$$

Rigid Body Model

Create load vector:

$$\vec{T} = \vec{P} \times \vec{F} = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} \times \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}$$



Calculate displacement vector:

$$\vec{q} = \vec{C}\vec{Q} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{bmatrix} \quad \begin{matrix} \vec{q}_{lin} \\ \vec{q}_{rot} \end{matrix}$$

$$\Delta\vec{P} = \vec{q}_{lin} + \vec{q}_{rot} \times \vec{P} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} + \begin{bmatrix} \Delta\theta_x \\ \Delta\theta_y \\ \Delta\theta_z \end{bmatrix} \times \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} \Delta P_x \\ \Delta P_y \\ \Delta P_z \end{bmatrix}$$

Rigid Body Model

Stiffness at Point *P*:

$$k_x = \frac{\sqrt{F_x^2 + F_y^2 + F_z^2}}{|\Delta P_x|} \qquad k_y = \frac{\sqrt{F_x^2 + F_y^2 + F_z^2}}{|\Delta P_y|} \qquad k_z = \frac{\sqrt{F_x^2 + F_y^2 + F_z^2}}{|\Delta P_z|}$$

Bearing Displacements

Determine bearing displacements:

Bearing 1: $$\Delta\vec{B}_1 = \vec{q}_{lin} + \vec{q}_{rot} \times \vec{B}_1 = \begin{bmatrix}\Delta x\\ \Delta y\\ \Delta z\end{bmatrix} + \begin{bmatrix}\Delta\theta_x\\ \Delta\theta_y\\ \Delta\theta_z\end{bmatrix} \times \begin{bmatrix}B_{1,x}\\ B_{1,y}\\ B_{1,z}\end{bmatrix} = \begin{bmatrix}\Delta B_{1,x}\\ \Delta B_{1,y}\\ \Delta B_{1,z}\end{bmatrix}$$

Bearing 2: $$\Delta\vec{B}_2 = \vec{q}_{lin} + \vec{q}_{rot} \times \vec{B}_2 = \begin{bmatrix}\Delta x\\ \Delta y\\ \Delta z\end{bmatrix} + \begin{bmatrix}\Delta\theta_x\\ \Delta\theta_y\\ \Delta\theta_z\end{bmatrix} \times \begin{bmatrix}B_{2,x}\\ B_{2,y}\\ B_{2,z}\end{bmatrix} = \begin{bmatrix}\Delta B_{2,x}\\ \Delta B_{2,y}\\ \Delta B_{2,z}\end{bmatrix}$$

Bearing 3: $$\Delta\vec{B}_3 = \vec{q}_{lin} + \vec{q}_{rot} \times \vec{B}_3 = \begin{bmatrix}\Delta x\\ \Delta y\\ \Delta z\end{bmatrix} + \begin{bmatrix}\Delta\theta_x\\ \Delta\theta_y\\ \Delta\theta_z\end{bmatrix} \times \begin{bmatrix}B_{3,x}\\ B_{3,y}\\ B_{3,z}\end{bmatrix} = \begin{bmatrix}\Delta B_{3,x}\\ \Delta B_{3,y}\\ \Delta B_{3,z}\end{bmatrix}$$

Bearing 4: $$\Delta\vec{B}_4 = \vec{q}_{lin} + \vec{q}_{rot} \times \vec{B}_4 = \begin{bmatrix}\Delta x\\ \Delta y\\ \Delta z\end{bmatrix} + \begin{bmatrix}\Delta\theta_x\\ \Delta\theta_y\\ \Delta\theta_z\end{bmatrix} \times \begin{bmatrix}B_{4,x}\\ B_{4,y}\\ B_{4,z}\end{bmatrix} = \begin{bmatrix}\Delta B_{4,x}\\ \Delta B_{4,y}\\ \Delta B_{4,z}\end{bmatrix}$$

Bearing Loads

Horizontal (=lateral) force

$$F_{B1,y} = F_{B1,h} = k_h \Delta B_{1,y}$$
$$F_{B2,y} = F_{B2,h} = k_h \Delta B_{2,y}$$
$$F_{B3,y} = F_{B3,h} = k_h \Delta B_{3,y}$$
$$F_{B4,y} = F_{B4,h} = k_h \Delta B_{4,y}$$

Vertical force

$$F_{B1,z} = F_{B1,v} = k_v \Delta B_{1,z}$$
$$F_{B2,z} = F_{B2,v} = k_v \Delta B_{2,z}$$
$$F_{B3,z} = F_{B3,v} = k_v \Delta B_{3,z}$$
$$F_{B4,z} = F_{B4,v} = k_h \Delta B_{4,z}$$

Check for brinelling:

Horizontal force of bearing *i* must not exceed the static load rating in the lateral direction:

$$\left|F_{Bi,h}\right| \le \frac{C_{o,l}}{f_w}$$

$C_{o,l}$ = static load rating in lateral direction
f_w = application factor

Bearing Loads

Positive vertical force of bearing *i* must not exceed the static load rating in the reverse radial direction:

$$F_{Bi,h+} \leq \frac{C_{o,rr}}{f_w}$$

$C_{o,rr}$ = static load rating in reverse radial direction

Negative vertical force of bearing *i* must not exceed the static load rating in the reverse radial direction:

$$\left|F_{Bi,h-}\right| \leq \frac{C_o}{f_w}$$

C_o = static load rating in radial direction

Equivalent dynamic load rating determines bearing life:

$$P_e = X\left|F_{Bi,v}\right| + Y\left|F_{Bi,h}\right|$$

Where *X* and *Y* are factors provided by the manufacturer that reflect the load inside the bearing as a result of a vertical and lateral load

Bearing Life

$$L = 100 \cdot \left(\frac{C_d}{f_w P_e}\right)^a$$

L = bearing life in km
C_d = dynamic load rating
f_w = application factor
a = exponent (a=3 for balls, 10/3 for rollers)

Resonance Frequencies

Equation of motion: $\vec{M}\ddot{\vec{q}} + \vec{K}\vec{q} = \vec{Q}$

For free vibration, solve homogenous solution: $\vec{Q} = \vec{0} \quad \rightarrow \vec{M}\ddot{\vec{q}} + \vec{K}\vec{q} = \vec{0}$

Assume harmonic solution:

$$u = U\cos(\omega t)$$

$$\frac{du}{dt} = \dot{u} = -U\omega\sin(\omega t)$$

$$\frac{du^2}{dt^2} = \ddot{u} = -U\omega^2\cos(\omega t) \quad \rightarrow \ddot{u} = -u\omega^2$$

Then: $\vec{M}\vec{q}\omega^2 + \vec{K}\vec{q} = \vec{0}$ and $\vec{M}\vec{q}\omega^2 + \vec{K}\vec{q} = \vec{0}$

Resonance Frequencies

$$\begin{bmatrix} k_s - m\omega^2 & 0 & 0 & 0 & -k_s h_s & 0 \\ 0 & 4k_h - m\omega^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4k_v - m\omega^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_v w^2 - I_x\omega^2 & 0 & 0 \\ -k_s h_s & 0 & 0 & 0 & k_v l^2 + k_s h_s^2 - I_y\omega^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & k_h l^2 - I_z\omega^2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The non-trivial solutions of this eigenvalue problem are found by calculating the determinate:

$$\det\left|K - M\omega^2\right| = 0$$

Here:

$$\left(k_h l^2 - I_z\omega^2\right)\left(k_v w^2 - m\omega^2\right)\left(4k_v - m\omega^2\right)\left(4k_h - m\omega^2\right)\left(I_y m\omega^4 - \left(k_s I_y + k_v l^2 m + k_s h_s^2 m\right)\omega^2 + k_s k_v l^2\right) = 0$$

Resonance Frequencies

Then:

$$\omega_a = \sqrt{\frac{k_h l^2}{I_z}}$$

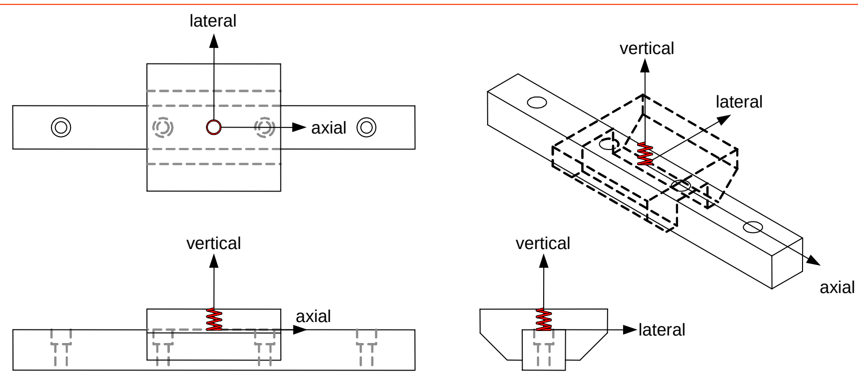
$$\omega_b = \sqrt{\frac{k_v w^2}{I_x}}$$

$$\omega_c = \sqrt{\frac{4k_v}{m}}$$

$$\omega_d = \sqrt{\frac{4k_h}{m}}$$

$$\omega_{e,f} = \sqrt{\frac{1}{2I_Y m}\left[k_s I_y + \left(k_v l^2 + k_s h_s^2\right)m \pm \sqrt{k_s^2\left(I_y^2 + h_s^4 m^2 + 2h_s^2 I_y m\right) + k_v^2 l^4 m^2 + 2k_s k_v l^2 m\left(h_s^2 m - I_y\right)}\right]}$$

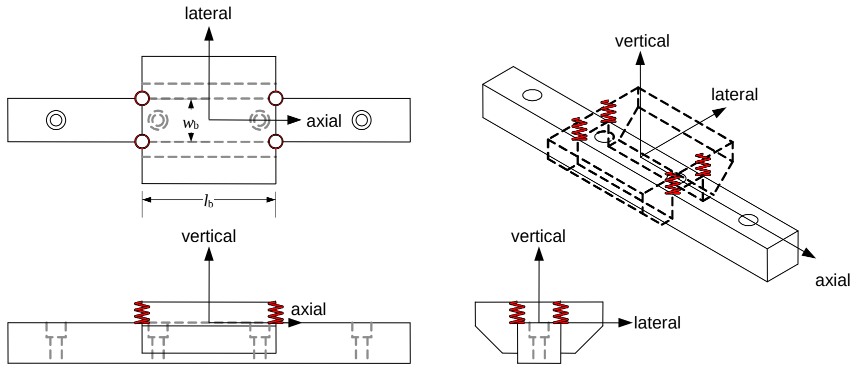
Using Springs – Single Springs



If no moment stiffness is known, use a single linear spring located in the center of the bearing. The spring connects the top surface of the rail with the top surface of the bearing.

$$k_{axial} = 0 \quad k_{lateral} = k_{lateral,literature} \quad k_{vertical} = k_{vertical,literature}$$

Using Springs – Multiple Springs

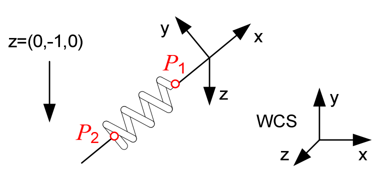


Use for better quality analysis and if stress concentrations are bothersome.

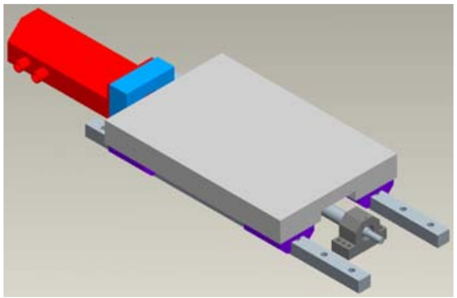
$$k_{axial} = 0 \quad k_{lateral} = \frac{k_{lateral,literature}}{4} \quad k_{vertical} = \frac{k_{vertical,literature}}{4}$$

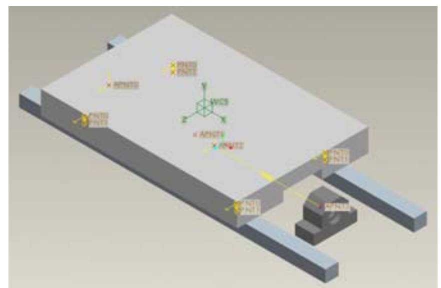
Modeling Springs in Pro/MECHANICA

- Bearings between axes are modeled as point-to-point springs.
- Point-to-point springs require two datum points (P_1, P_2) that define the length and position of the spring.
- The vector between the two datum points defines the springs' x-axis.
- The z-axis of the spring is defined as a vector relative to the World Coordinate System (WCS).
- Springs that attach to solids require the rotational DOF's of the datum points to be constrained.



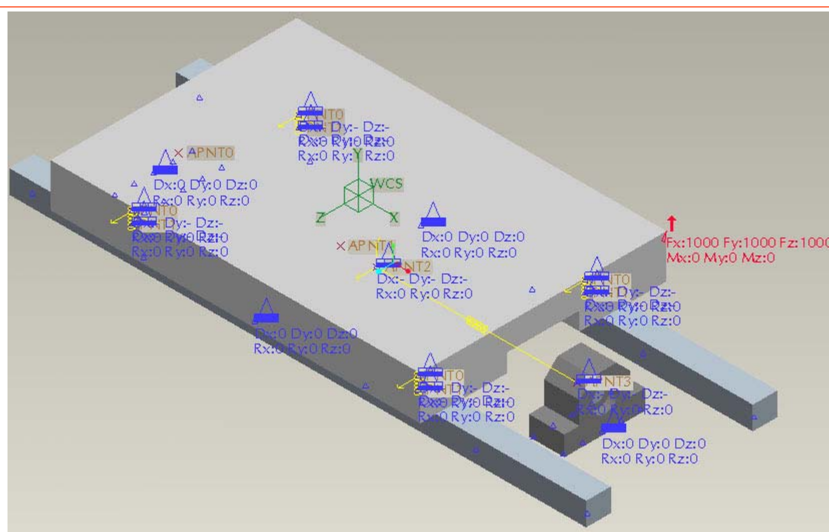
Linear Axis with 4 Bearings and Ball Screw



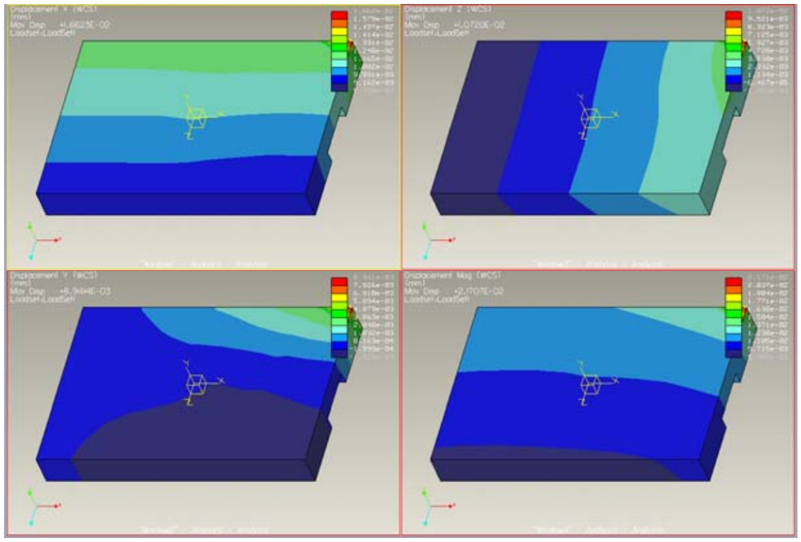


Suppress non-relevant components (motor, motor mount).
Suppress non-relevant features (bolt holes).
Replace bearings and ball screw with springs.

Linear Axis with Springs and Constraints



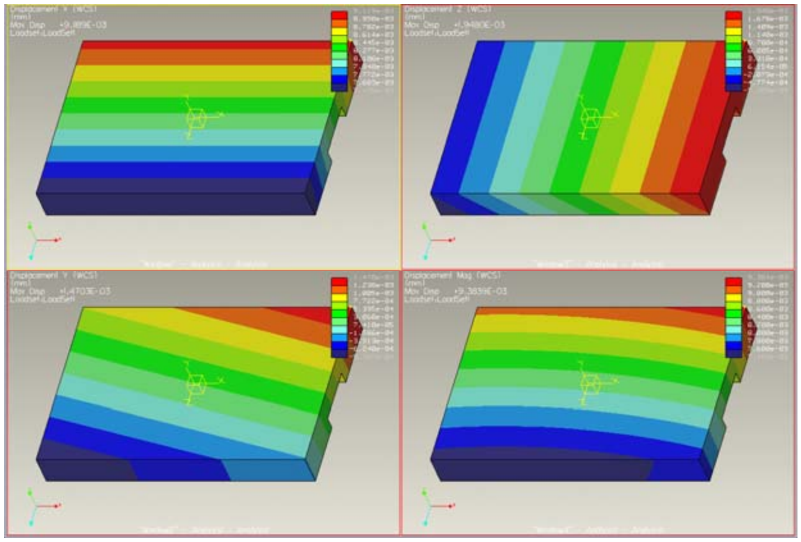
Linear Axis with Springs and Steel



Comparison

	Rigid Body Model	**FEA with Springs and Steel Table**	
Disp in x [μm]	8.07	16.6	+106%
Disp in y [μm]	1.91	10.7	+460%
Disp in z [μm]	1.45	8.94	+517%

Linear Axis with Springs and Stiff-Stuff



Comparison

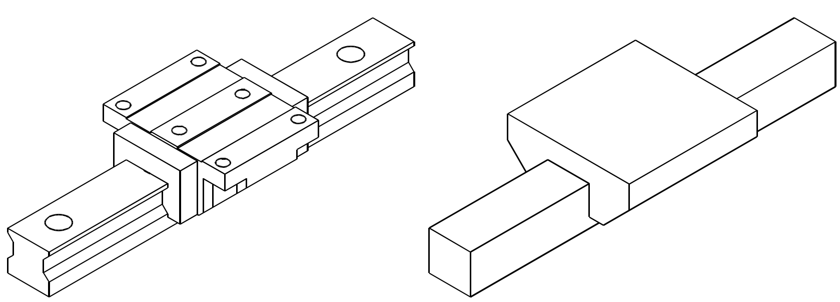
	Rigid Body Model	FEA with Springs and Steel Table		FEA with Springs and Steel from Stiff-Stuff	
Disp in x [μm]	8.07	16.6	+106%	9.12	+13%
Disp in y [μm]	1.91	10.7	+460%	1.95	+2.1%
Disp in z [μm]	1.45	8.94	+517%	1.47	+1.3%

Modeling Bearings as Solids

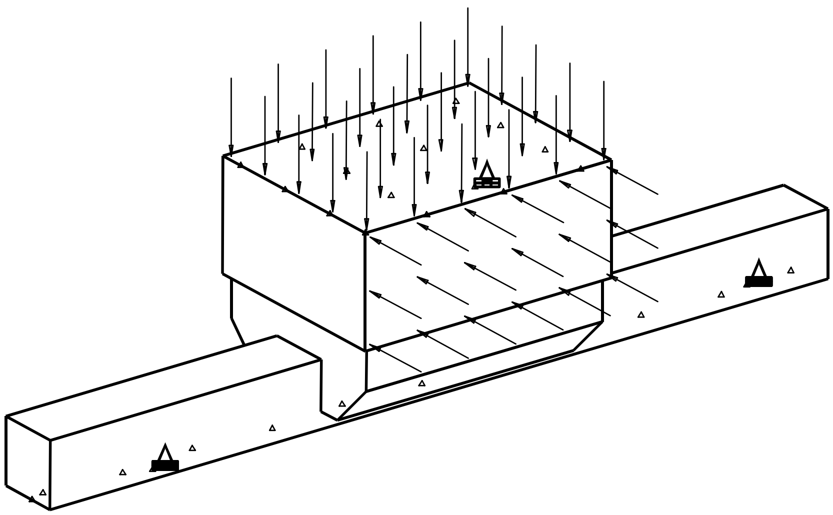
- Modeling bearings as solids has the advantage of having a model that looks realistic.
- Avoids the replacing of the bearings with springs (tedious and error-prone).
- BUT: Analysis time will significantly increase and convergence might be an issue.

Technique

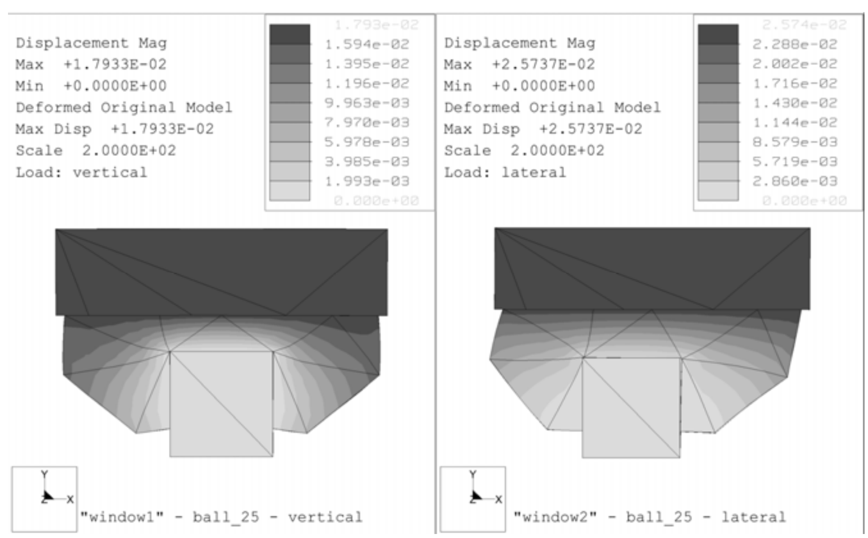
- Identify a Young's modulus that will allow the solid bearing to deform just as the real bearing would under the same loading.
- Remove unnecessary details that will increase analysis time.



Define FEA Analysis that Mimics a Vertical and Lateral Load



FEA Results



Create a Modulus Table

Type	Size	Preload	Length [mm]	Width [mm]	Height [mm]	$E_{vertical}$ [N/mm^2]	$E_{lateral}$ [N/mm^2]
Ball	25	8%	81	70	29.5	3650	2040
Ball	35	8%	105	100	40	3560	2100
Ball	45	8%	133	120	50	3690	2180
Ball	55	8%	159	140	57	3505	2370
Roller	25	13%	91	70	30	5400	2920
Roller	35	13%	114	100	41	7060	3650
Roller	45	13%	140	120	51	8500	4060
Roller	55	13%	166.5	140	58	8800	4180