Given: Standard British golf ball:

$$M = 45.9 \pm 0.3\,g \quad (20 \text{ to } 1)$$
$$D = 41.1 \pm 0.3\,mm \quad (20 \text{ to } 1)$$

Find: (a) Density and specific gravity
(b) Estimate of uncertainties in calculated values.

Solution:

Density is mass per unit volume, so

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{3}{4\pi}\frac{M}{(D/2)^3} = \frac{6}{\pi}\frac{M}{D^3}$$

$$\rho = \frac{6}{\pi} \times 0.0459\,kg \times \frac{1}{(0.0411)^3\,m^3} = 1260\,kg/m^3$$

and
$$SG = \frac{\rho}{\rho_{H_2O}} = 1260\,\frac{kg}{m^3} \times \frac{m^3}{1000\,kg} = 1.26$$

The uncertainty in density is given by

$$u_\rho = \pm\left[\left(\frac{M}{\rho}\frac{\partial\rho}{\partial M}u_M\right)^2 + \left(\frac{D}{\rho}\frac{\partial\rho}{\partial D}u_D\right)^2\right]^{1/2}$$

$$\frac{M}{\rho}\frac{\partial\rho}{\partial M} = \frac{M}{\rho}\frac{1}{V} = \frac{V}{V} = 1 \quad ; \quad u_M = \pm\frac{0.3}{45.9} = \pm 0.654\,\%$$

$$\frac{D}{\rho}\frac{\partial\rho}{\partial D} = \frac{D}{\rho}\left(-3\,\frac{6}{\pi}\frac{M}{D^4}\right) = -3\left(\frac{6M}{\pi D^3 \rho}\right) = -3$$

$$u_D = \pm\frac{0.3}{41.1} = 0.730\,\%$$

Thus

$$u_\rho = \pm\left[(u_M)^2 + (-3u_D)^2\right]^{1/2} = \pm\left\{(0.654)^2 + [-3(0.730)]^2\right\}^{1/2}$$

$$u_\rho = \pm 2.29\,\% \quad (\pm 28.9\,kg/m^3)$$

$$u_{SG} = u_\rho = \pm 2.29\,\% \quad (\pm 0.0289)$$

Summarizing

$$\rho = 1260 \pm 28.9\,kg/m^3 \quad (20 \text{ to } 1) \qquad \rho$$

$$SG = 1.26 \pm 0.0289 \quad (20 \text{ to } 1) \qquad SG$$