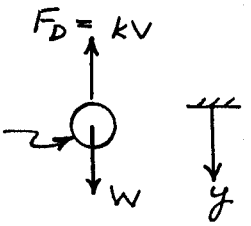
Given: Small particle accelerating from rest in a fluid.
Net weight is W, resisting force $F_D = kV$, where V is speed.

Find: Time required to reach 95 percent of terminal speed, V_t.

Solution: Consider the particle to be a system. Apply Newton's second law.



Basic equation: $\Sigma F_y = ma_y$

Assumptions: (1) W is net weight
(2) Resisting force acts opposite to V

Then

$$\Sigma F_y = W - kV = ma_y = m\frac{dV}{dt} = \frac{W}{g}\frac{dV}{dt}$$

or $$\frac{dV}{dt} = g\left(1 - \frac{k}{W}V\right)$$

Separating variables,

$$\frac{dV}{1 - \frac{k}{W}V} = g\,dt$$

Integrating, noting that velocity is zero initially,

$$\int_0^V \frac{dV}{1-\frac{k}{W}V} = -\frac{W}{k}\ln\left(1-\frac{k}{W}V\right)\Big]_0^V = \int_0^t g\,dt = gt$$

or

$$1 - \frac{k}{W}V = e^{-\frac{kgt}{W}} \quad ; \quad V = \frac{W}{k}\left[1 - e^{-\frac{kgt}{W}}\right]$$

But $V \to V_t$ as $t \to \infty$, so $V_t = \frac{W}{k}$. Therefore

$$\frac{V}{V_t} = 1 - e^{-\frac{kgt}{W}}$$

When $\frac{V}{V_t} = 0.95$, then $e^{-\frac{kgt}{W}} = 0.05$ and $\frac{kgt}{W} = 3$. Thus

$$t = 3W/gk$$