Problem 12.18

Given: Reversible, adiabatic flow of air from a large tank through a converging nozzle discharging to atmosphere

$T_0 = 600\,K$

$p_0 = 600\,kPa$

$p_b = p_{atm} = 101\,kPa$

$A_t = 1.29 \times 10^{-3}\,m^2$

Find: (a) range of tank pressure, p_0, for which $M_t = 1.0$
(b) $\dot{m}$ for conditions given

Solution:

Basic equations: $\dot{m} = \rho V A = \text{const}$ $\qquad p = \rho R T$

Computing equations: $\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \qquad \frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/k-1}$

Assumptions: (1) steady flow (2) isentropic flow in nozzle (3) uniform flow at a section (4) ideal gas

The nozzle will be choked, i.e. $M_t = 1.0$ for $p_b/p_0 \le 0.528$

Since $p_b = 101\,kPa$, nozzle is choked for

$$p_0 \ge \frac{p_b}{0.528} = \frac{101\,kPa}{0.528} = 191\,kPa$$ ← p_0

Thus for $p_0 = 600\,kPa$, $M_t = 1.0$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \;;\quad T_t = \frac{T_0}{1 + \frac{k-1}{2} M_t^2} = \frac{600\,K}{1 + 0.2(1.0)^2} = 500\,K$$

$$V_t = M_t c_t = M_t (kRT_t)^{1/2} = 1.0\left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 500\,K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 448\,m/s$$

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2} M^2\right]^{k/k-1} \;;\quad p_t = \frac{p_0}{\left[1 + \frac{k-1}{2} M_t^2\right]^{k/k-1}} = \frac{600\,kPa}{\left[1 + 0.2(1.0)^2\right]^{3.5}} = 317\,kPa$$

$$\rho_t = \frac{p_t}{RT_t} = 317 \times 10^3 \frac{N}{m^2} \times \frac{kg \cdot K}{287\,N \cdot m} \times \frac{1}{500\,K} = 2.21\,kg/m^3$$

Finally,

$$\dot{m} = \rho_t V_t A_t = 2.21 \frac{kg}{m^3} \times 448 \frac{m}{s} \times 1.29 \times 10^{-3}\,m^2 = 1.28\,kg/s$$ ← $\dot{m}$