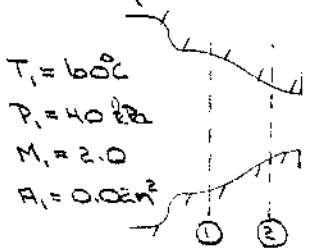
Problem 12.5

Given: Steady, isentropic flow of air through a passage



(passage shape unspecified)

Find: M_2, shape of passage

Solution:

Basic equations: $h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$

Assumptions: (1) steady flow
(2) isentropic flow
(3) uniform flow at a section
(4) $\Delta g = 0$
(5) ideal gas

$M_2 = \frac{V_2}{c_2}$ where $c_2 = (kRT_2)^{1/2}$. Hence T_2 must be found

$h_2 = h_1 + \frac{1}{2}(V_1^2 - V_2^2)$

$V_1 = M_1 c_1 = M_1 (kRT_1)^{1/2} = 2.0 \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 333 K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 732$ m/s

$T_2 = T_1 + \frac{1}{2c_p}(V_1^2 - V_2^2)$

$= 333K + \left[(732)^2 - (519)^2\right] \frac{m^2}{s^2} \times \frac{1}{2} \times \frac{kg \cdot K}{1.0 \times 10^3 J} \times \frac{N \cdot s^2}{kg \cdot m}$

$T_2 = 466$ K

$c_2 = (kRT_2)^{1/2} = \left(1.4 \times 287 \frac{N \cdot m}{kg \cdot K} \times 466 K \times \frac{kg \cdot m}{N \cdot s^2}\right)^{1/2} = 432$ m/s

$M_2 = \frac{V_2}{c_2} = \frac{519}{432} = 1.20$ ← M_2

Since $M_2 < M_1$ and $M_2 > 1.0$, then passage from ① to ② is a supersonic diffuser as shown above