

Given: Velocity field in  $xy$  plane,  $\vec{V} = a\hat{i} + bx\hat{j}$ , where  
 $a = 2 \text{ m/s}$  and  $b = 1 \text{ s}^{-1}$ .

Find: (a) Equation for streamline through  $(x, y) = (2, 5)$ .

(b) At  $t = 2 \text{ s}$ , coordinates of particle  $(0, 4)$  at  $t = 0$ .

(c) At  $t = 3 \text{ s}$ , coordinates of particle  $(1, 4.25)$  at  $t = 1 \text{ s}$ .

(d) Compare pathline, streamline, streakline.

Solution: For a streamline  $\frac{dx}{u} = \frac{dy}{v}$

For  $\vec{V} = a\hat{i} + bx\hat{j}$ ,  $u = a$  and  $v = bx$ , so  $\frac{dx}{a} = \frac{dy}{bx}$  or

$$x dx = \frac{a}{b} dy$$

Integrating

$$\frac{x^2}{2} = \frac{a}{b} y + C' \quad \text{or} \quad y = \frac{b}{2a} x^2 + C$$

Evaluating  $C$  at  $(x, y) = (2, 5)$ ,

$$C = y - \frac{b}{2a} x^2 = 5 \text{ m} - \frac{1}{2} \times \frac{1}{\text{s}} \times \frac{\text{s}}{2 \text{ m}} (2 \text{ m})^2 = 4 \text{ m}$$

Streamline through  $(x, y) = (2, 5)$  is  $y = \frac{x^2}{4} + 4$  (a)

To locate particles, derive parametric equations

$$u_p = \frac{dx}{dt} = a, \quad dx = a dt, \quad \text{and} \quad x - x_0 = a(t - t_0)$$

$$v_p = \frac{dy}{dt} = bx, \quad dy = bx dt = b(x_0 + at - at_0)$$

$$y - y_0 = bx_0(t - t_0) + \frac{a}{2}(t^2 - t_0^2) - at_0(t - t_0)$$

For the particle at  $(x_0, y_0) = (0, 4)$  at  $t = 0$ ,

$$x = 0 + at$$

$$\text{so at } t = 2 \text{ s, } x = \frac{2 \text{ m}}{\text{s}} \times 2 \text{ s} = 4 \text{ m}$$

$$y = 4 + \frac{at^2}{2}$$

$$\text{so at } t = 2 \text{ s, } y = 4 + \frac{1}{2} \times \frac{2 \text{ m}}{\text{s}} \times (2)^2 \text{ s}^2$$

$$y = 8 \text{ m}$$

(b)

For the particle at  $(x, y) = (1, 4.25)$  at  $t = 1$  s,

$$x = x_0 + a(t - t_0) = 1 + a(t - 1)$$

$$\text{so at } t = 3 \text{ s, } x = 1 + 2 \frac{m}{s} (3 - 1) = 5 \text{ m}$$

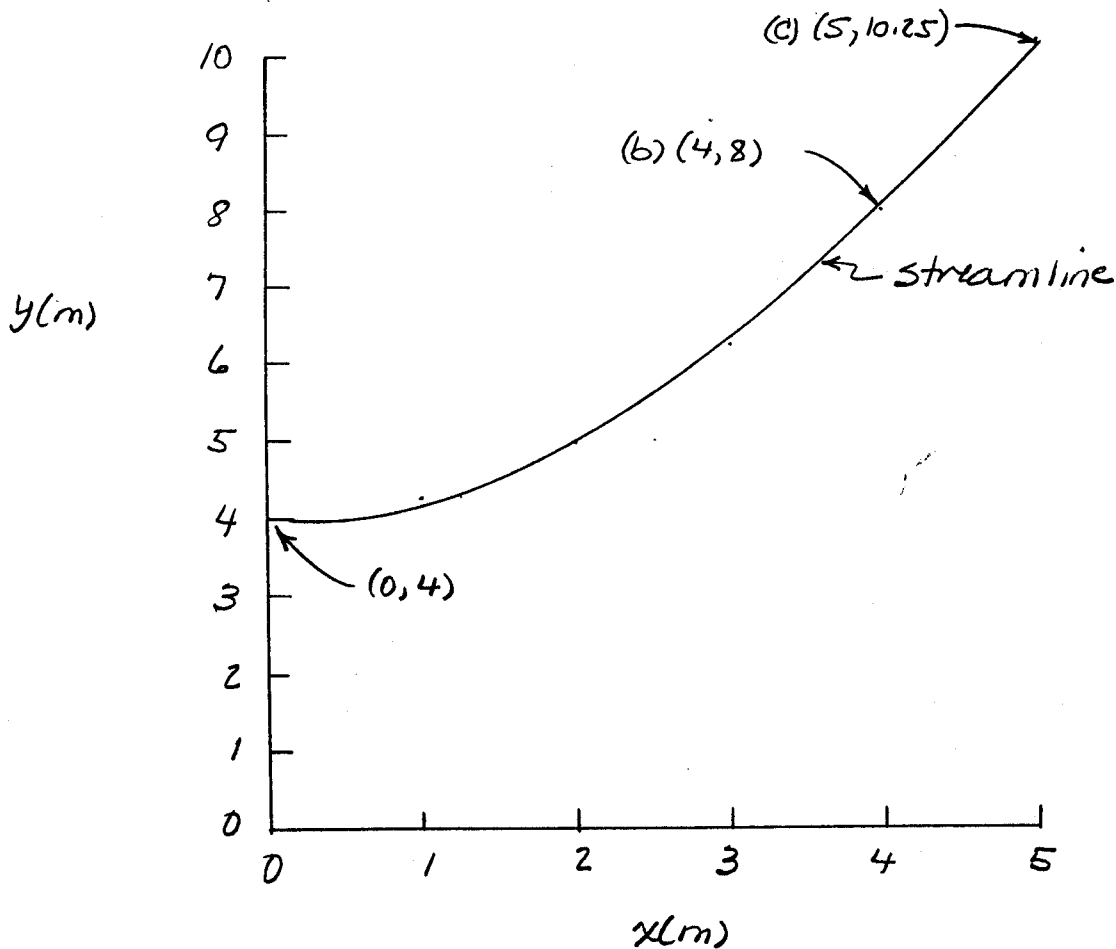
$$y = y_0 + bx_0(t - t_0) + \frac{a}{2}(t^2 - t_0^2) - at_0(t - t_0)$$

$$= 4.25 + \frac{1}{3} \times 1 \text{ m} \times (t - 1) + \frac{1}{2} \times 2 \frac{m}{s} (t^2 - 1) - 2 \frac{m}{s} \times 1 \text{ s} (t - 1)$$

$$\text{so at } t = 3 \text{ s, } y = 4.25 + 2 + 8 - 4 = 10.25 \text{ m}$$

(c)

All these points lie on the same streamline, as shown below:



For this steady flow, streamlines, pathlines, and streaklines coincide, as expected.