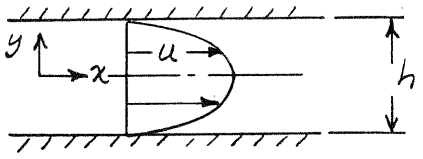
Problem 2.30

Given: Laminar flow between parallel plates.

$$\frac{u}{u_{max}} = 1 - \left(\frac{2y}{h}\right)^2$$



$T = 15°C$, $u_{max} = 0.05$ m/s, $h = 1$ mm, water

Find: Force on $A = 0.1$ m² section of lower plate.

Solution: Apply definitions of Newtonian fluid, shear stress.

Basic equations: $\tau = \frac{F}{A}$, $\tau_{yx} = \mu \frac{du}{dy}$

Assumptions: (1) Newtonian fluid

From the given profile, $u = u_{max}\left[1 - \left(\frac{2y}{h}\right)^2\right]$, so $\frac{du}{dy} = u_{max}(-2)\left(\frac{2y}{h}\right)\left(\frac{2}{h}\right)$

$$= -\frac{8 u_{max} y}{h^2}$$

At lower surface, $y = -h/2$

$$\tau_{yx}(\text{lower}) = \mu \frac{du}{dy}\Big)_{y=-h/2} = \mu\left[-\frac{8 u_{max}(-h/2)}{h^2}\right] = \frac{4\mu u_{max}}{h}$$

$\tau_{yx} > 0$ and surface is positive, so to right.

$$F = \tau_{yx} A = \frac{4\mu u_{max} A}{h}$$

From Appendix A, Table A.8, $\mu = 1.14 \times 10^{-3}$ N·s/m at 15°C, so

$$F = 4 \times 1.14 \times 10^{-3} \frac{N \cdot s}{m^2} \times 0.05 \frac{m}{s} \times 0.1\ m^2 \times \frac{1}{5\ mm} \times 10^3 \frac{mm}{m}$$

$F = 0.228$ N (to right)

F ←