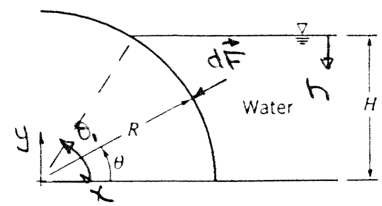
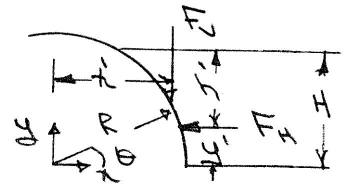
Given: Curved surface, in shape of quarter cylinder, with radius $R = 0.750\,m$ and width $w = 3.55\,m$; water stands to depth $H = 0.650\,m$



Find: Magnitude and line of action of:
(a) vertical force, and
(b) horizontal force
on the curved surface.



Solution:

Basic equations: $\frac{dp}{dh} = \rho g$, $F_V = \int p\,dA_y$, $x' F_V = \int x\,dF_V$

Computing equations: $F_H = p_c A$, $h' = h_c + \frac{I_{xx}}{h_c A}$

Assumptions: (1) static liquid (2) ρ = constant
(3) p_{atm} acts at free surface of the water

Then on integrating $dp = \rho g\,dh$, we obtain $p = \rho g h$

From the geometry $h = H - R\sin\theta$, $y = R\sin\theta$, $x = R\cos\theta$

$\theta_1 = \sin^{-1} H/R$, $dA = wR\,d\theta$

$$F_V = \int p\,dA_y = \int \rho g h\,dA\sin\theta = \int_0^{\theta_1} \rho g (H - R\sin\theta)\sin\theta\, wR\,d\theta$$

$$F_V = \rho g w R \int_0^{\theta_1} (H\sin\theta - R\sin^2\theta)\,d\theta = \rho g w R\left[-H\cos\theta - R\left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)\right]_0^{\theta_1}$$

$$F_V = \rho g w R\left[H(1-\cos\theta_1) - R\left(\frac{\theta_1}{2} - \frac{\sin 2\theta_1}{4}\right)\right] \quad (1)$$

Evaluating for $\theta_1 = \sin^{-1}\frac{H}{R} = \sin^{-1}\frac{0.650}{0.750} = 60°\ (\pi/3)$.

$$F_V = 999\frac{kg}{m^3} \times 9.81\frac{m}{s^2} \times 3.55\,m \times 0.75\,m\left[0.65\,m(1-\cos 60°) - 0.75\,m\left(\frac{\pi}{6} - \frac{\sin 120°}{4}\right)\right]\frac{N\cdot s^2}{kg\cdot m}$$

$F_V = 2.47\,kN$ ← F_V

$$x' F_V = \rho g w R \int_0^{\theta_1} R\cos\theta(H\sin\theta - R\sin^2\theta)\,d\theta = \rho g w R^2 \int_0^{\theta_1} (H\sin\theta\cos\theta - R\sin^2\theta\cos\theta)\,d\theta$$

$$x' F_V = \rho g w R^2\left[H\frac{\sin^2\theta}{2} - R\frac{\sin^3\theta}{3}\right]_0^{\theta_1}$$

$$x' = \frac{\rho g w R^2}{F_V}\left[\frac{H}{2}\sin^2\theta_1 - \frac{R}{3}\sin^3\theta_1\right] \quad (2)$$

$$x' = 999\frac{kg}{m^3} \times 9.81\frac{m}{s^2} \times 3.55\,m \times (0.75\,m)^2 \times \frac{1}{2.47\times10^3\,N}\left[\frac{0.650\,m}{2}\sin^2 60° - \frac{0.750\,m}{3}\sin^3 60°\right]\frac{N\cdot s^2}{kg\cdot m}$$

$x' = 0.645\,m$ ← x'

$$F_H = p_c A = \rho g h_c H w = \rho g \frac{H}{2} H w = \frac{\rho g H^2 w}{2} \quad (3)$$

$$F_H = \frac{1}{2} \times 999\frac{kg}{m^3} \times 9.81\frac{m}{s^2} \times (0.65\,m)^2 \times 3.55\,m \times \frac{N\cdot s^2}{kg\cdot m} = 7.35\,kN$$ ← F_H

$$h' = h_c + \frac{I_{xx}}{h_c A} = h_c + \frac{\frac{1}{12} W H^3}{h_c A} = \frac{H}{2} + \frac{1}{12}\frac{WH^3}{\frac{H}{2} H W} = \frac{H}{2} + \frac{H}{6} = \frac{2}{3} H$$

$$y' = H - h' = H - \frac{2}{3} H = \frac{1}{3} H \qquad (4)$$

$$y' = \frac{1}{3} H = \frac{1}{3} \times 0.650\ \text{m} = 0.217\ \text{m} \longleftarrow y'$$

The computing equations for the plot are:

$$\theta_1 = \sin^{-1} H/R$$

$$F_V = \rho g w R^2 \left[\frac{H}{R}(1 - \cos\theta_1) - \frac{\theta_1}{2} + \frac{\sin 2\theta_1}{4} \right]$$

$$x' = \frac{\rho g w R^3 \sin^2\theta_1}{F_V} \left[\frac{1}{2}\frac{H}{R} - \frac{1}{3}\sin\theta_1 \right]$$

$$F_H = \frac{\rho g H^2 w}{2}$$

$$y' = \frac{H}{3}$$