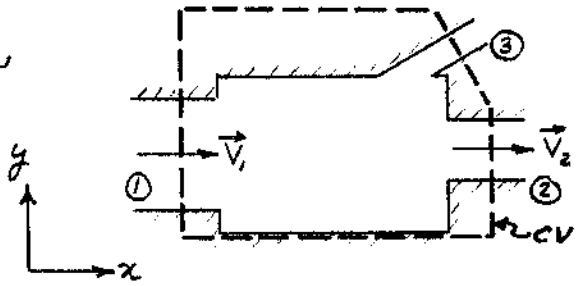
Problem 4.17

Given: Steady, incompressible flow through device shown.



$A_1 = 1\,ft^2$; $A_2 = 0.5\,ft^2$; $A_3 = 0.2\,ft^2$

$\vec{V}_1 = 10\hat{i}\ ft/s$, $\vec{V}_2 = 30\hat{i}\ ft/s$

Find: Volume flow rate through port 3.

Solution: Apply conservation of mass to CV shown

Basic equation: $0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V}\cdot d\vec{A}$ ($\frac{\partial}{\partial t}\int_{CV} \rho\, d\forall = 0$ (1))

Assumptions: (1) Steady flow
(2) Uniform flow at each section
(3) Incompressible flow, ρ = constant

Then $0 = \vec{V}_1\cdot\vec{A}_1 + \vec{V}_2\cdot\vec{A}_2 + \vec{V}_3\cdot\vec{A}_3$

or $0 = -|V_1 A_1| + |V_2 A_2| + \vec{V}_3\cdot\vec{A}_3$ (flow in at ①, out at ②)

Solving,

$$\vec{V}_3\cdot\vec{A}_3 = |V_1 A_1| - |V_2 A_2|$$

$$\vec{V}_3\cdot\vec{A}_3 = \left|10\tfrac{ft}{s} \times 1\,ft^2\right| - \left|30\tfrac{ft}{s} \times 0.5\,ft^2\right| = -5.00\tfrac{ft^3}{s}$$

Therefore

$$Q_3 = \vec{V}_3\cdot\vec{A}_3 = -5.00\,ft^3/s$$ (minus sign means into CV) ← Q_3