Problem 4.185

Given: Centrifugal water pump operation under conditions as follows:
\[ P_1 = P_2 = 4 \text{ in} \quad Q = 300 \text{ gpm} \]
\[ P_1 = 8 \text{ in Hg (vacuum)} \quad P_2 = 35 \text{ psig} \quad z_1 = z_2 \]

Input = 9.1 hp

Find: pump efficiency

Solution: Apply the energy equation to the CV shown. Neglect all losses to find the energy added to the fluid

Basic equations:
\[ \eta = \frac{W_{in}}{W_{out}} \quad \text{where } W_{in} = \text{power into fluid} \]
\[ W_{shear} = \int_{CV} \tau \text{ d}A \quad W_{fric} = \int_{CV} f \text{ d}x \]
\[ W_{oth} = \int_{CV} \left( \frac{1}{2} \rho V^2 \right) \text{ d}x \]

Assumptions:
1. \( \rho = 0 \)
2. \( W_{shear} = 0 \) (by choice of CV); \( W_{fric} = 0 \)
3. Steady flow
4. Neglected Du
5. Incompressible flow
6. Uniform flow at inlet and outlet

Then
\[ W_{in} = \left( \rho_1 v_1 + \frac{v_1^2}{2} \right) \left( -\frac{n}{g} \right) + \left( \rho_2 v_2 + \frac{v_2^2}{2} \right) \left( \frac{n}{g} \right) \]

Since \( v_1 = v_2 \) (from continuity)
\[ W_{in} = \rho_1 \left( \rho_2 v_2 - \rho_1 v_1 \right) = Q \left( P_2 - P_1 \right) \]

\[ P_1 = \rho g h = 56 \text{ psi} \]

\[ P_1 = 13.6 \times 1.94 \frac{\text{slug}}{\text{ft}^3} \times 32.2 \frac{\text{ft}^2}{\text{sl}} \times (-8 \text{ in}) \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{1 \text{ slug}}{1 \text{ lb}} \times \frac{1 \text{ ft}}{1 \text{ ft}} \times \frac{1 \text{ lb} \cdot \text{ft}}{1 \text{ hp} \cdot \text{sec}} \]

\[ P_1 = -3.93 \text{ psi} \]

\[ W_{in} = 300 \text{ gal} \times \frac{\text{ft}^3}{10 \text{ gal}} \times \min \times \left[ 35 - (-3.93) \right] \times \frac{1 \text{ hp} \cdot \text{sec}}{550 \text{ ft} \cdot \text{lb}} \]

\[ W_{in} = -6.81 \text{ hp} \quad \text{(negative sign indicates energy added to fluid)} \]

Then
\[ \eta = \frac{W_{in}}{9.1} = 0.714 \text{ or } 71.4 \text{ percent} \]