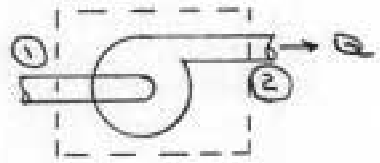
Problem 4.185

Given: Centrifugal water pump operating under conditions as follows:



$D_1 = D_2 = 4\text{ in.}$ $\quad Q = 300\text{ gpm}$

$p_1 = 8\text{ in Hg (vacuum)}, \quad p_2 = 35\text{ psig} \qquad z_1 = z_2$

$\mathcal{P}_{input} = 9.1\text{ hp}$

Find: pump efficiency.

Solution: Apply the energy equation to the CV shown. Neglect all losses to find the energy added to the fluid.

Basic equations: $\eta = \dfrac{\dot{W}_s}{\mathcal{P}_{in}}$ where $\dot{W}_s$ = power into fluid

$$\underbrace{\dot{Q}}_{=0(1)} - \dot{W}_s - \underbrace{\dot{W}_{shear}}_{=0(2)} - \underbrace{\dot{W}_{other}}_{=0(2)} = \underbrace{\frac{\partial}{\partial t}\int_{CV} e\rho\, d\forall}_{=0(3)} + \int_{CS}\Big(\underbrace{u}_{=0(4)} + pv + \frac{V^2}{2} + \underbrace{gz}_{=0(5)}\Big)\rho \vec{V}\cdot d\vec{A}$$

Assumptions: (1) $\dot{Q} = 0$
(2) $\dot{W}_{shear} = 0$ (by choice of CV); $\dot{W}_{other} = 0$
(3) steady flow
(4) neglect Δu
(5) $\Delta z = 0$
(6) incompressible flow
(7) uniform flow at inlet and outlet

Then $\quad -\dot{W}_s = \left(p_1 v_1 + \dfrac{V_1^2}{2}\right)\{-\dot{m}\} + \left(p_2 v_2 + \dfrac{V_2^2}{2}\right)\{\dot{m}\}$

Since $\dot{m} = \rho Q$ and $V_1 = V_2$ (from continuity)

$$-\dot{W}_s = \rho Q(p_2 v_2 - p_1 v_1) = Q(p_2 - p_1)$$

$p_1 = \rho g h = SG\,\rho_{H_2O}\, g h$

$$p_1 = 13.6 \times 1.94\,\frac{\text{slug}}{\text{ft}^3} \times 32.2\,\frac{\text{ft}}{\text{s}^2} \times (-8\text{ in}) \times \frac{\text{ft}}{12\text{ in}} \times \frac{\text{lbf}\cdot\text{s}^2}{\text{ft}\cdot\text{slug}} \times \frac{\text{ft}^2}{144\text{ in}^2} = -3.93\text{ psig}$$

$$\therefore -\dot{W}_s = 300\,\frac{\text{gal}}{\text{min}} \times \frac{\text{ft}^3}{7.48\text{ gal}} \times \frac{\text{min}}{60\text{ s}} \times [35 - (-3.93)]\frac{\text{lbf}}{\text{in}^2} \times \frac{144\text{ in}^2}{\text{ft}^2} \times \frac{\text{hp}\cdot\text{s}}{550\text{ ft}\cdot\text{lbf}}$$

$\dot{W}_s = -6.81\text{ hp}$ (negative sign indicates energy added).

Then

$$\eta = \frac{\dot{W}_s}{\mathcal{P}_{in}} = \frac{6.81}{9.1} = 0.748 \text{ or } 74.8 \text{ percent} \longleftarrow \eta$$