Problem 4.40

Given: Steady flow of water past a porous flat plate. Suction is constant. Velocity profile at section cd is

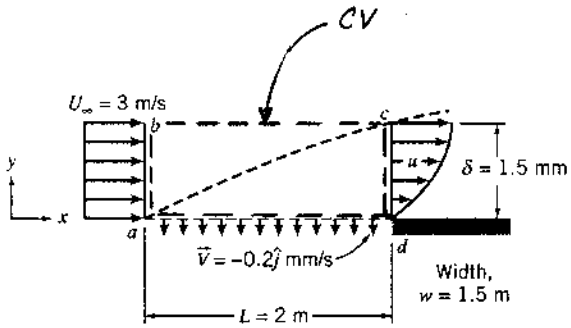
$$\frac{u}{U_\infty} = 3\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^{1.5}$$

Find: Mass flow rate across Section bc.

Solution: Apply conservation of mass using the CV shown.

Basic equation:

$$0 = \frac{\partial}{\partial t}\int_{cv} \rho\, d\forall + \int_{cs} \rho \vec{V}\cdot d\vec{A}$$



Assumptions: (1) Steady flow
(2) Incompressible flow
(3) $\vec{V} = -v_0\hat{j}$ along da

Then

$$0 = \int_{cs} \rho \vec{V}\cdot d\vec{A} = \int_{ab} \rho \vec{V}\cdot d\vec{A} + \dot{m}_{bc} + \int_{cd} \rho \vec{V}\cdot d\vec{A} + \int_{da} \rho \vec{V}\cdot d\vec{A}$$

or

$$0 = -\rho U_\infty w\delta + \dot{m}_{bc} + \int_0^\delta \rho U_\infty\left[3\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^{1.5}\right] w\, dy + \rho v_0 w L$$

Thus

$$\dot{m}_{bc} = \rho U_\infty w\delta - \rho U_\infty w\delta \int_0^1 \left[3\left(\frac{y}{\delta}\right) - 2\left(\frac{y}{\delta}\right)^{1.5}\right] d\left(\frac{y}{\delta}\right) - \rho v_0 w L$$

$$= \rho w\left\{U_\infty\delta - U_\infty\delta\left[\frac{3}{2}\left(\frac{y}{\delta}\right)^2 - \frac{2}{2.5}\left(\frac{y}{\delta}\right)^{2.5}\right]_0^1 - v_0 L\right\}$$

$$= \rho w\left[U_\infty\delta - U_\infty\delta\left(\frac{3}{2} - \frac{2}{2.5}\right) - v_0 L\right] = \rho w(0.3 U_\infty\delta - v_0 L)$$

$$= 999\ \frac{kg}{m^3} \times 1.5\ m\left(0.3 \times 3\ \frac{m}{s} \times 0.0015\ m - 0.0002\ \frac{m}{s} \times 2\ m\right)$$

$$\dot{m}_{bc} = 1.42\ kg/s \quad (\dot{m} > 0, \text{ so } \underline{\text{out}} \text{ of CV})$$

$\dot{m}_{bc}$ ←