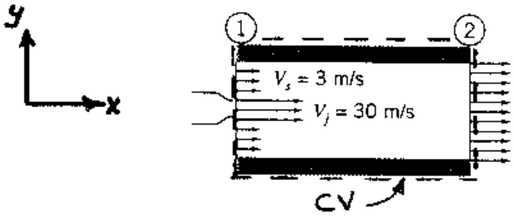
Problem 4.72

Given: Water jet pump as shown in the sketch.

$A_j = 0.01\,m^2$ $\qquad A_2 = 0.075\,m^2$



The two streams are thoroughly mixed at section ②, and the inlet pressures are the same.

Find: (a) The velocity at the pump exit
(b) The pressure rise, $p_2 - p_1$

Solution: Apply continuity and the x component of momentum to the inertial CV shown.

Basic equations:

$$0 = \frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho \vec{V}\cdot d\vec{A} \quad (=0(1))$$

$$F_{Sx} + F_{Bx} = \frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall + \int_{CS} u\rho \vec{V}\cdot d\vec{A} \quad (F_{Bx}=0(5),\ \tfrac{\partial}{\partial t}=0(1))$$

Assumptions: (1) Steady flow
(2) Incompressible flow
(3) Uniform flow at each section
(4) No viscous forces act on CV
(5) $F_{Bx} = 0$

Then from continuity

$$0 = \{-|\rho V_s A_s|\} + \{-|\rho V_j A_j|\} + \{|\rho V_2 A_2|\} = -\rho V_s A_s - \rho V_j A_j + \rho V_2 A_2$$

$$V_2 = \frac{1}{A_2}(V_s A_s + V_j A_j) \quad ; \quad A_s = A_2 - A_j = (0.075 - 0.01)\,m^2 = 0.065\,m^2$$

$$V_2 = \frac{1}{0.075\,m^2}\left(3\frac{m}{s}\times 0.065\,m^2 + 30\frac{m}{s}\times 0.01\,m^2\right) = 6.60\,\frac{m}{s}$$

V_2

and

$$p_1 A_2 - p_2 A_2 = u_s\{-|\rho V_s A_s|\} + u_j\{-|\rho V_j A_j|\} + u_2\{|\rho V_2 A_2|\}$$

$$u_s = V_s \qquad u_j = V_j \qquad u_2 = V_2$$

$$\Delta p = p_2 - p_1 = \frac{1}{A_2}\left(+\rho V_s^2 A_s + \rho V_j^2 A_j - \rho V_2^2 A_2\right) = \frac{\rho}{A_2}\left(+V_s^2 A_s + V_j^2 A_j - V_2^2 A_2\right)$$

$$= 999\frac{kg}{m^3}\times\frac{1}{0.075\,m^2}\left[(3.0)^2(0.065) + (30)^2(0.01) - (6.6)^2(0.075)\right]\frac{m^2}{s^2}\times m^2 \times \frac{N\cdot s^2}{kg\cdot m}$$

$$p_2 - p_1 = 84.2\ kPa$$

$p_2 - p_1$