Problem 5.42

Given: Incompressible, two-dimensional flow field with \( \nu = 0 \), has a \( y \)-component of velocity given by

\[ v = -Ay \]

where units of \( u \) are m/s; \( x \) and \( y \) are in meters and \( A \) is a dimensional constant.

Find: (a) the dimensions of the constant \( A \)
(b) the simplest \( x \)-component of velocity for this flow field.
(c) the acceleration of a fluid particle at the point \((x, y) = (1, 2)\)

Solution:

(a) Since \( v = -Ay \), then the dimensions of \( A \), \([A]\), are given by

\[ [A] = \left[ \frac{L}{T L L} \right] = \frac{1}{T} \]

(b) Apply the continuity equation for the conditions given.

Basic equation:

\[ \frac{\partial u}{\partial t} + \frac{\partial (u^2)}{\partial x} + \frac{\partial (uv)}{\partial y} = 0 \]

For incompressible flow, \( \frac{\partial \rho}{\partial t} = 0 \). Thus with \( \nu = 0 \), the basic equation reduces to

\[ \frac{\partial u}{\partial t} + \frac{\partial (u^2)}{\partial x} + \frac{\partial (uv)}{\partial y} = 0 \]

Then,

\[ \frac{\partial u}{\partial t} = -\frac{\partial}{\partial x} (-Ay) = Ay \]

and

\[ u(x, t) = \int_{-\infty}^{x} Ay \, dx = \frac{1}{2} A x^2 + f(y) \]

The simplest \( x \)-component of velocity is obtained with \( f(y) = 0 \)

\[ u(x, t) = \frac{1}{2} A x^2 \]

(c) The acceleration of a fluid particle is given by

\[ \mathbf{a}_p = \frac{\partial \mathbf{v}}{\partial t} = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \]

\[ \mathbf{a}_p = \frac{1}{2} A x^2 \left[ \frac{1}{2} A x^2 - Ay \right] - Ax^2 \frac{\partial y}{\partial x} \left[ \frac{1}{2} A x^2 - Ay \right] \]

At the point \((x, y) = (1, 2)\)

\[ \mathbf{a}_p = \frac{1}{2} A^2 \left( 1 \right)^2 \frac{1}{2} A^2 \left( 2 \right)^2 \]

\[ \mathbf{a}_p = \frac{1}{2} A^2 \left[ \frac{1}{2} \right] = \frac{1}{2} A^2 \]

\[ \mathbf{a}_p = \mathbf{a} \]