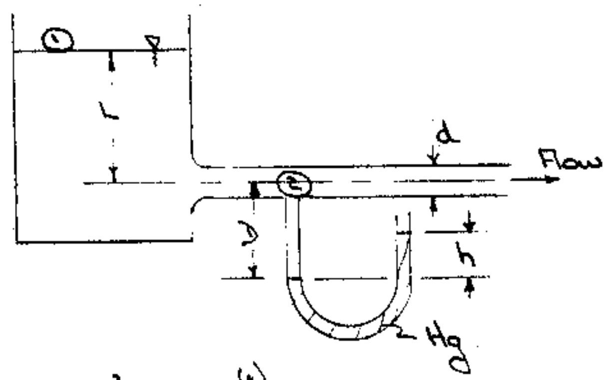
Problem 6.45

Given: Water flow from a large tank as shown.

$L = 12\ ft \quad D = 2\ ft \quad d = 2\ in$

$h = 6"$



Find: (a) Velocity in discharge pipe
(b) Rate of discharge

Solution:

Basic equations: $\frac{p_1}{\rho} + \cancel{\frac{V_1^2}{2}}^{\approx 0(5)} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + \cancel{gz_2}^{0(8)}$

$Q = \int u\,dA$

Assumptions:
(1) steady flow
(2) incompressible flow
(3) no friction
(4) flow along a streamline
(5) $V_1 \approx 0$, ie. large tank
(6) $p_1 = p_{atm}$
(7) uniform flow at section 2
(8) $z_2 = 0$

From the Bernoulli equation, $V_2 = \left[2\frac{(p_1 - p_2)}{\rho} + 2gL\right]^{1/2} = \left[2\frac{(p_{atm} - p_2)}{\rho} + 2gL\right]^{1/2}$

From the conditions of the manometer,

$p_{atm} + \gamma_{Hg}h - \gamma_{H_2O}D = p_2 \quad$ and $\quad p_{atm} - p_2 = \gamma_{H_2O}D - \gamma_{Hg}h$

Substituting into the expression for V_2,

$V_2 = \left[\frac{2}{\rho}(\gamma_{H_2O}D - \gamma_{Hg}h) + 2gL\right]^{1/2} = \left[\frac{2}{\rho}\gamma_{H_2O}(D - SG_{Hg}h) + 2gL\right]^{1/2} = \left[2g(D - SG_{Hg}h + L)\right]^{1/2}$

$V_2 = \left[2 \times 32.2\frac{ft}{s^2} \times \left(2ft - 13.6 \times \frac{1}{2}ft + 12ft\right)\right]^{1/2} = 21.5\ ft/s$ ← V_2

$Q = \int u\,dA = V_2A_2 \quad$ (for uniform flow at 2)

$Q = V_2\frac{\pi d^2}{4} = 21.5\frac{ft}{s} \times \frac{\pi}{4} \times \left(\frac{2}{12}\right)^2 ft^2 = 0.469\ ft^3/s$ ← Q