Problem 7.19

Given: Power, $\mathscr{P}$, required to drive a fan depends on ρ, Q, D and ω.

Find: Dependence of $\mathscr{P}$ on other parameters.

Solution: Apply Buckingham Π procedure.

① $\mathscr{P} \quad \rho \quad Q \quad D \quad \omega$ $\qquad n = 5$ parameters

② Choose primary dimensions M, L, t

③ $\mathscr{P} \quad \rho \quad Q \quad D \quad \omega$

$\frac{ML^2}{t^3} \quad \frac{M}{L^3} \quad \frac{L^3}{t} \quad L \quad \frac{1}{t}$ $\qquad r = 3$ primary dimensions

④ $\rho, D, \omega \qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 2$ dimensionless groups will result. Setting up dimensional equations,

$$\Pi_1 = \rho^a D^b \omega^c \mathscr{P} = \left(\frac{M}{L^3}\right)^a (L)^b \left(\frac{1}{t}\right)^c \left(\frac{ML^2}{t^3}\right) = M^0 L^0 t^0$$

Summing exponents,

$M: \; a + 1 = 0 \qquad a = -1$

$L: \; -3a + b + 2 = 0 \qquad b = -5$

$t: \; -c - 3 = 0 \qquad c = -3$

$$\therefore \; \Pi_1 = \frac{\mathscr{P}}{\rho D^5 \omega^3}$$

$$\Pi_2 = \rho^d D^e \omega^f Q = \left(\frac{M}{L^3}\right)^d (L)^e \left(\frac{1}{t}\right)^f \left(\frac{L^3}{t}\right) = M^0 L^0 t^0$$

Summing exponents,

$M: \; d + 0 = 0 \qquad d = 0$

$L: \; -3d + e + 3 = 0 \qquad e = -3$

$t: \; -f - 1 = 0 \qquad f = -1$

$$\therefore \; \Pi_2 = \frac{Q}{D^3 \omega}$$

⑥ Check using primary dimensions F, L, t

$$\Pi_1 = \frac{FL}{t} \frac{L^4}{Ft^2} \frac{1}{L^5} t^3 = [1] \checkmark \qquad \Pi_2 = \frac{L^3}{t} \frac{1}{L^3} t = [1] \checkmark$$

Thus $\Pi_1 = f(\Pi_2)$, or

$$\frac{\mathscr{P}}{\rho D^5 \omega^3} = f\left(\frac{Q}{D^3 \omega}\right)$$

$\mathscr{P}$