Given: A model (to scale) of a tractor-trailer rig is tested in a wind tunnel; $A_m = 1.08$ ft². For $V_m = 250$ ft/s, $F_{Dm} = 76.3$ lbf.

Find: (a) drag coefficient for the model.
(b) F_{D_p} at $V_p = 55$ mi/hr if $C_{D_p} = C_{D_m}$
(c) V_m if $V_p = 55$ mi/hr
(d) Is answer to part (c) reasonable.

Solution:

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

For the model assuming air at STP,

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A} = \frac{76.3 \text{ lbf}}{0.5 \times 0.002377 \frac{\text{slug}}{\text{ft}^3} \times (250)^2 \text{ ft}^2 \times 1.08 \text{ ft}^2} \times \frac{1}{\text{ft}^2} \times \frac{\text{slug} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2}$$

$$C_{Dm} = 0.951 \quad\underleftarrow{\hspace{3cm}} \quad C_{Dm}$$

$$F_{D_p} = \frac{1}{2}\rho V_p^2 A_p C_{D_p} \qquad C_{D_p} = C_{DA} = 0.951 \qquad A_p = \left(\frac{L_p}{L_m}\right)^2 A_m = 100\, A_m$$

$$F_{D_p} = \frac{1}{2} \times 0.002377 \frac{\text{slug}}{\text{ft}^3} \times \left(55 \frac{\text{mi}}{\text{hr}} \times 5280 \frac{\text{ft}}{\text{mi}} \times \frac{\text{hr}}{3600\text{s}}\right)^2 \times 100 \times 1.08\,\text{ft}^2 \times 0.951 \times \frac{\text{lbf} \cdot \text{s}^2}{\text{ft} \cdot \text{slug}}$$

$$F_{D_p} = 794 \text{ lbf} \quad\underleftarrow{\hspace{3cm}}\quad F_{D_p}$$

For dynamic similarity between model and prototype

$$\left(\frac{\rho V L}{\mu}\right)_m = \left(\frac{\rho V L}{\mu}\right)_p \qquad \text{or} \qquad V_m = V_p \frac{\rho_p}{\rho_m} \frac{L_p}{L_m} \frac{\mu_m}{\mu_p} = V_p \times 1 \times 10 \times 1$$

$$V_m = 10\, V_p = 550 \text{ mi/hr} \quad\underleftarrow{\hspace{3cm}}\quad V_m$$

$$V_m = 550 \frac{\text{mi}}{\text{hr}} \times 5280 \frac{\text{ft}}{\text{mi}} \times \frac{\text{hr}}{3600\text{s}} = 807 \text{ ft/s}$$

For air at standard conditions, the speed of sound, $c = \sqrt{kRT}$

$$c = \left(1.4 \times 53.3 \frac{\text{ft} \cdot \text{lbf}}{\text{lbm} \cdot \text{R}} \times 519\,\text{R} \times 32.2 \frac{\text{lbm}}{\text{slug}} \times \frac{\text{slug} \cdot \text{ft}}{\text{lbf} \cdot \text{s}^2}\right)^{1/2} = 1117 \text{ ft/s}$$

$$M = \frac{V}{c} = \frac{807}{1117} = 0.72$$

At this value of M, compressibility would be important in the model test. Thus, the speed is not practical. $\underleftarrow{\hspace{2cm}}$