Given: Thrust and torque of propeller depend on D, ω, V, μ, ρ

Model: $D = 600$ mm, $\omega = 2000$ rpm, $V = 45$ m/s, $F_t = 110$ N (thrust), $T = 10$ N·m

Prototype: $D = 6$ m, $\omega = ?$, $V = 120$ m/s, $F_t = ?$, $T = ?$

Find: (a) ω, (b) F_t and (c) T for prototype, neglecting effects of viscosity, under dynamically similar conditions.

<u>Solution</u>: There are two problems here. (1) Determine $F_t = f_1(D, \omega, V, \mu, \rho)$ and (2) $T = f_2(D, \omega, V, \mu, \rho)$. Since μ is to be ignored, do not select it as a repeating parameter. Instead, select D, ω, ρ as repeating variables.

(1) $F_t = f_1(D, \omega, V, \mu)$

① $F_t \quad D \quad \omega \quad V \quad \mu \quad \rho \qquad n = 6$ parameters

② Select F, L, t as primary dimensions.

③

F_t	D	ω	V	μ	ρ
F	L	$\frac{1}{t}$	$\frac{L}{t}$	$\frac{Ft}{L^2}$	$\frac{Ft^2}{L^4}$

$r = 3$ primary dimensions

④ Choose $D, \omega, V \qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 3$ dimensionless groups will result. Setting up dimensional equations,

$$\Pi_1 = D^a \omega^b \rho^c F_t = (L)^a \left(\frac{1}{t}\right)^b \left(\frac{Ft^2}{L^4}\right)^c F = F^0 L^0 t^0$$

F: $c + 1 = 0 \qquad c = -1$
L: $a - 4c = 0 \qquad a = -4$
t: $-b + 2c = 0 \qquad b = -2$

$$\Pi_1 = \frac{F_t}{\rho \omega^2 D^4}$$

$$\Pi_2 = D^a \omega^b \rho^c V = (L)^a \left(\frac{1}{t}\right)^b \left(\frac{Ft}{L^4}\right)^c \frac{L}{t} = F^0 L^0 t^0$$

F: $c = 0 \qquad c = 0$
L: $a - 4c + 1 = 0 \qquad a = -1$
t: $-b + c - 1 = 0 \qquad b = -1$

$$\Pi_2 = \frac{V}{\omega D}$$

$$\Pi_3 = D^a \omega^b \rho^c \mu = (L)^a \left(\frac{1}{t}\right)^b \left(\frac{Ft^2}{L^4}\right)^c \frac{Ft}{L^2} = F^0 L^0 t^0$$

F: $c + 1 = 0 \qquad c = -1$
L: $a - 4c - 2 = 0 \qquad a = 4c + 2 = -2$
t: $-b + 2c + 1 = 0 \qquad b = 2c + 1 = -1$

$$\Pi_3 = \frac{\mu}{\rho \omega D^2}$$

Then $\Pi_1 = f_1(\Pi_2, \Pi_3)$ or $\dfrac{F_t}{\rho \omega^2 D^4} = f_1\left(\dfrac{V}{\omega D}, \dfrac{\mu}{\rho \omega D^2}\right)$

If viscous effects are neglected, then $\Pi_1 = g_1(\Pi_2)$ or $\frac{F_t}{\rho\omega^2 D^4} = g_1\left(\frac{V}{\omega D}\right)$

For dynamic similarity, $\Pi_2)_{model} = \Pi_2)_{prototype}$, or

$$\frac{V_m}{\omega_m D_m} = \frac{V_p}{\omega_p D_p}$$

Thus $\omega_p = \omega_m \frac{V_p}{V_m}\frac{D_m}{D_p} = (2000\,rpm)\left(\frac{120}{45}\right)\left(\frac{1}{10}\right) = 533\ rpm$ ω_p

When $\Pi_2)_{model} = \Pi_2)_{prototype}$, then neglecting μ, $\Pi_1)_{model} = \Pi_1)_{prototype}$, or

$$\frac{F_{tm}}{\rho_m\omega_m^2 D_m^4} = \frac{F_{tp}}{\rho_p\omega_p^2 D_p^4}$$; assume $\rho_m = \rho_p$

Then $F_{tp} = F_{tm}\left(\frac{\omega_p}{\omega_m}\right)^2\left(\frac{D_p}{D_m}\right)^4 = 110\,N \cdot \left(\frac{533}{2000}\right)^2 (10)^4 = 78.1\ kN$ F_{tp}

(2) The analysis of Π_2 and Π_3 for the second problem is identical to that for problem (1). Combining T with D, ω and ρ gives

$$\Pi_4 = D^a\omega^b\rho^c T = (L)^a\left(\frac{1}{t}\right)^b\left(\frac{Ft^2}{L^4}\right)^c(FL) = M^0L^0t^0$$

$$\left.\begin{array}{lll} F: & c+1=0 & c=-1 \\ L: & a-4c+1=0 & a=4c-1=-5 \\ t: & -b+2c=0 & b=2c=-2 \end{array}\right\} \quad \Pi_4 = \frac{T}{\rho\omega^2D^5}$$

Thus $\Pi_4 = f_2(\Pi_2, \Pi_3)$ or neglecting μ, $\Pi_4 = g_2(\Pi_2)$. For dynamic similarity, $\Pi_4)_{model} = \Pi_4)_{prototype}$, or

$$\frac{T_m}{\rho_m\omega_m^2 D_m^5} = \frac{T_p}{\rho_p\omega_p^2 D_p^5}$$; assume $\rho_m = \rho_p$

Then $T_p = T_m\left(\frac{\omega_p}{\omega_m}\right)^2\left(\frac{D_p}{D_m}\right)^5 = 10\,N\cdot m \cdot \left(\frac{533}{2000}\right)^2(10)^5 = 71\ kN\cdot m$ T_p

6 Check, using M, L, t:

$$\Pi_1 = \frac{ML}{t^2}\frac{L^3}{M}\frac{t^2}{1}\frac{1}{L^4} = [1] \checkmark$$

$$\Pi_2 = \frac{L}{t}\frac{t}{1}\frac{1}{L} = [1] \checkmark$$

$$\Pi_3 = \frac{M}{Lt}\frac{L^3}{M}\frac{t}{1}\frac{1}{L^2} = [1] \checkmark$$

$$\Pi_4 = \frac{ML^2}{t^2}\frac{L^3}{M}\frac{t^2}{1}\frac{1}{L^5} = [1] \checkmark$$