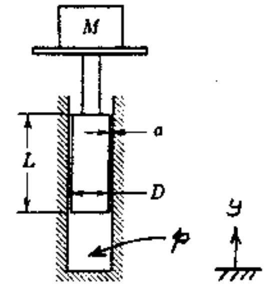
Problem 8.15

**Given:** Piston-cylinder device, as shown.

$D = 6\ mm \quad L = 25\ mm$

Liquid is SAE 30 oil at 20°C.




**Find:** (a) $M$ to develop $p = 1.5$ MPa (gage)
(b) Leakage flow rate in terms of $a$
(c) Maximum $a$ to provide < 1 mm/min movement.

**Solution:** The mass may be found from a force balance on the piston.

$$\Sigma F_y = \frac{\pi D^2}{4}(p - p_{atm}) - Mg = 0 \quad \text{so} \quad M = \frac{\pi D^2}{4g} p_{gage}$$

$$M = \frac{\pi}{4} \times (0.006)^2 m^2 \times 1.5 \times 10^6 \frac{N}{m^2} \times \frac{s^2}{9.81\,m} \times \frac{kg \cdot m}{N \cdot s^2} = 4.32\ kg$$ M

The leakage flow rate may be evaluated for flow between flat plates. From Eq. 8.6c, neglecting motion of the piston,

$$\frac{Q}{l} = \frac{a^3 \Delta p}{12 \mu L} \quad \text{or, since } l = \pi D, \quad Q = \frac{\pi}{12} \frac{a^3 \Delta p D}{\mu L} \sim a^3$$ Q

The piston, moving downward at speed, $v$, displaces liquid at rate

$$Q = \frac{\pi D^2}{4} v = \frac{\pi}{4}(0.006)^2 m^2 \times 0.001 \frac{m}{min} \times \frac{min}{60\,s} = 4.71 \times 10^{-10} m^3/s.$$

Then, with $\mu = 0.42\ N \cdot sec/m^2$ (at 20°C, Fig. A.2),

$$a = \left[\frac{12 \mu Q L}{\pi D \Delta p}\right]^{1/3} = \left[\frac{12}{\pi} \times 0.42 \frac{N \cdot s}{m^2} \times 4.71 \times 10^{-10} \frac{m^3}{s} \times 0.025\,m \times \frac{1}{0.006\,m} \times \frac{m^2}{1.5 \times 10^6 N}\right]^{1/3}$$

$$a = 1.28 \times 10^{-5} m \quad (12.8\ \mu m)$$ a

Check assumptions: $\bar{V} = \frac{Q}{A} = \frac{Q}{\pi D a} = \frac{1}{\pi} \times 4.71 \times 10^{-10} \frac{m^3}{s} \times \frac{1}{0.006\,m} \times \frac{1}{1.28 \times 10^{-5} m} = 1.95 \frac{mm}{s}$

Thus $\frac{v}{\bar{V}} = 1 \frac{mm}{min} \times \frac{sec}{1.95\,mm} \times \frac{min}{60\,s} = 0.00855 < 0.01$

Therefore piston motion is negligible.

Also $Re = \frac{\bar{V} a}{\nu}$ ; $\nu = \frac{\mu}{\rho} = \frac{\mu}{SG\,\rho_{H_2O}}$. From Table A.2 (Appendix A), SG = 0.92

$$\nu = 0.42 \frac{N \cdot s}{m^2} \times \frac{m^3}{(0.92)1000\,kg} \times \frac{kg \cdot m}{N \cdot s^2} = 4.57 \times 10^{-4}\ m^2/s.$$

$$Re = 1.95 \times 10^{-3} \frac{m}{s} \times 1.28 \times 10^{-5} m \times \frac{s}{4.57 \times 10^{-4}\ m^2} = 5.46 \times 10^{-5} \lll 1$$

Therefore flow is surely laminar!