Problem *9.26

Given: Numerical results of Blasius for laminar boundary-layer flow.

Find: Plot v/U versus y/δ for $Re_x = 10^5$.

Solution: For the Blasius solution, $\psi = \sqrt{U\nu x}\, f(\eta)$ and $\eta = y\sqrt{\frac{U}{\nu x}}$

From the stream function, $v = -\frac{\partial \psi}{\partial x} = -\left[\frac{1}{2}\sqrt{\frac{\nu U}{x}} f(\eta) + \sqrt{U\nu x}\,\frac{df}{d\eta}\frac{\partial \eta}{\partial x}\right]$

But $\frac{\partial \eta}{\partial x} = -\frac{1}{2}\frac{y}{x}\sqrt{\frac{U}{\nu x}} = -\frac{1}{2}\frac{\eta}{x}$

Thus $v = -\frac{1}{2}\sqrt{\frac{U\nu}{x}} f(\eta) - \sqrt{U\nu x}\,\frac{df}{d\eta}\left(-\frac{1}{2}\frac{\eta}{x}\right) = \frac{1}{2}\sqrt{\frac{U\nu}{x}}\left[\eta f'(\eta) - f(\eta)\right]$

and $\frac{v}{U} = \frac{1}{2}\sqrt{\frac{\nu}{Ux}}\left[\eta f'(\eta) - f(\eta)\right] = \frac{\eta f'(\eta) - f(\eta)}{2\sqrt{Re_x}}$ ← $\frac{v}{U}$

Also $\frac{y}{\delta} = \frac{y}{5\sqrt{\frac{\nu x}{U}}} = \frac{\eta}{5}$

Tabulate from Table 9.1:

η	$\eta f'(\eta) - f(\eta)$	v/U
0	0	0
0.4	0.0265	4.20×10^{-5}
1.0	0.164	2.60×10^{-4}
1.4	0.316	4.99×10^{-4}
2.0	0.610	9.64×10^{-4}
2.4	0.827	1.31×10^{-3}
3.0	1.14	1.80×10^{-3}
3.4	1.32	2.09×10^{-3}
4.0	1.52	2.40×10^{-3}
4.4	1.60	2.53×10^{-3}
5.0	1.67	2.65×10^{-3}

Plot: