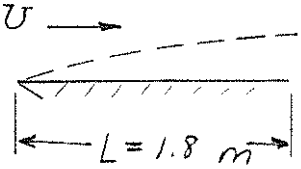
Problem 9.35

Given: Laminar boundary layer in std. air, sinusoidal profile.
Conditions: $U = 3.2$ m/s, $L = 1.8$ m, $b = 0.9$ m.



Find: Plot δ, δ^*, and τ_w vs. x/L.

Solution: Apply momentum integral equation for steady, incompressible flow.

Computing equations: $\tau_w = \rho U^2 \frac{d\theta}{dx}$, $\frac{\delta^*}{\delta} = \int_0^1 (1 - \frac{u}{U}) d\lambda$, $\frac{\theta}{\delta} = \int_0^1 \frac{u}{U}(1 - \frac{u}{U}) d\lambda$

For the sinusoidal profile,

$$\frac{\theta}{\delta} = \int_0^1 \sin\tfrac{\pi}{2}\lambda\,(1 - \sin\tfrac{\pi}{2}\lambda)\,d\lambda = \int_0^1 (\sin\tfrac{\pi}{2}\lambda - \sin^2\tfrac{\pi}{2}\lambda)\,d\lambda$$

$$= \int_0^1 (\sin\tfrac{\pi}{2}\lambda - \frac{1-\cos\pi\lambda}{2})\,d\lambda = \left[-\frac{2}{\pi}\cos\tfrac{\pi}{2}\lambda - \frac{\lambda}{2} + \frac{1}{2\pi}\sin\pi\lambda\right]_0^1$$

$$\frac{\theta}{\delta} = -\frac{2}{\pi}(0-1) - \frac{1}{2} + \frac{1}{2\pi}(0-0) = \frac{2}{\pi} - \frac{1}{2} = \frac{4-\pi}{2\pi} = 0.137$$

Also $\tau_w = \mu \left.\frac{du}{dy}\right)_{y=0} = \mu U \left.\frac{d}{dy}\sin(\frac{\pi}{2}\frac{y}{\delta})\right]_{y=0} = \frac{\pi\mu U}{2\delta}\left.(\cos\frac{\pi}{2}\frac{y}{\delta})\right]_{y=0} = \frac{\pi\mu U}{2\delta}$

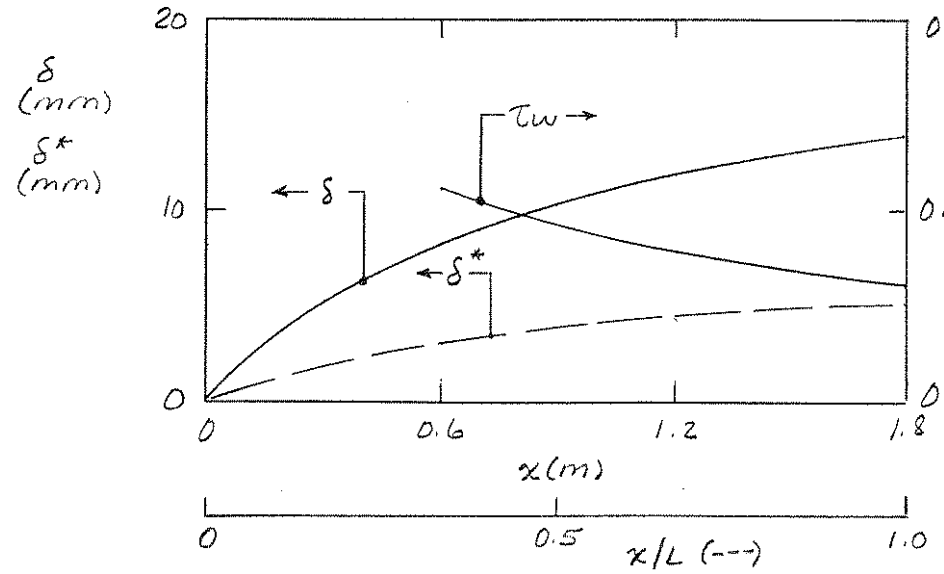
Thus $\tau_w = \rho U^2 \frac{d\theta}{dx} = \frac{\pi\mu U}{2\delta} = \rho U^2 (\frac{4-\pi}{2\pi})\frac{d\delta}{dx}$ or $\delta\,d\delta = \frac{\pi^2}{4-\pi}\frac{\mu}{\rho U}dx$

Integrating, $\frac{\delta^2}{2} = \frac{\pi^2}{4-\pi}\frac{\mu}{\rho U}x$ so $\frac{\delta}{x} = \sqrt{\frac{2\pi^2}{4-\pi}}\sqrt{\frac{\mu}{\rho U x}} = \frac{4.80}{\sqrt{Re_x}}$ ← δ/x

Also $\frac{\delta^*}{\delta} = \int_0^1 (1 - \frac{u}{U})\,d\lambda = \int_0^1 (1 - \sin\frac{\pi}{2}\lambda)\,d\lambda = (\lambda + \frac{2}{\pi}\cos\frac{\pi}{2}\lambda)\Big|_0^1 = 1 - \frac{2}{\pi} = 0.363$ ← δ^*/δ

Tabulate: $Re_x = \frac{Ux}{\nu} = 3.2\frac{m}{s} \cdot x\,m \times \frac{s}{1.46\times10^{-5}m^2} = 2.19\times10^5 x$

x (m)	Re_x (---)	$1/\sqrt{Re_x}$ (---)	δ (mm)	δ^* (mm)	τ_w (N/m²)
0.6	1.31×10^5	0.00276	7.95	2.89	0.0114
1.2	2.63×10^5	0.00195	11.2	4.07	0.00804
1.8	3.94×10^5	0.00159	13.7	4.97	0.00656



$\tau_w = \frac{\pi}{2}\frac{\mu}{\rho U \delta}\rho U^2$

$= \frac{1}{2}\rho U^2 \frac{\mu}{\rho U x}\frac{x}{\delta}\pi$

$\tau_w = \frac{\pi}{4.80}\frac{1}{\sqrt{Re_x}}\frac{1}{2}\rho U^2$

Plot ←