Problem 9.9

Given: Linear, parabolic, and sinusoidal velocity profiles for laminar boundary layer.

(a) linear $u/U = y/\delta$

(b) parabolic $u/U = 2\frac{y}{\delta} - \left(\frac{y}{\delta}\right)^2$

(c) sinusoidal $u/U = \sin\frac{\pi}{2}\frac{y}{\delta}$

Find: ratio δ^*/δ for each profile.

<u>Solution:</u>

Definition: $\delta^* = \int_0^\infty \left(1-\frac{u}{U}\right)dy \approx \int_0^\delta \left(1-\frac{u}{U}\right)dy$ (9.1)

Then, $\frac{\delta^*}{\delta} = \frac{1}{\delta}\int_0^\delta \left(1-\frac{u}{U}\right)dy = \int_0^1 \left(1-\frac{u}{U}\right)d\left(\frac{y}{\delta}\right) = \int_0^1 \left(1-\frac{u}{U}\right)d\eta$

(a) Linear profile $u/U = y/\delta = \eta$

$\delta^*/\delta = \int_0^1 \left(1-\frac{u}{U}\right)d\eta = \int_0^1 (1-\eta)\,d\eta = \left[\eta - \frac{1}{2}\eta^2\right]_0^1 = \frac{1}{2}$

(b) Parabolic profile $u/U = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 = 2\eta - \eta^2$

$\delta^*/\delta = \int_0^1 \left(1-\frac{u}{U}\right)d\eta = \int_0^1 (1-2\eta+\eta^2)\,d\eta = \left[\eta - \eta^2 + \frac{1}{3}\eta^3\right]_0^1 = \frac{1}{3}$

(c) Sinusoidal profile $u/U = \sin\frac{\pi}{2}\frac{y}{\delta} = \sin\frac{\pi}{2}\eta$

$\delta^*/\delta = \int_0^1 \left(1-\frac{u}{U}\right)d\eta = \int_0^1 \left(1-\sin\frac{\pi}{2}\eta\right)d\eta = \left[\eta + \frac{2}{\pi}\cos\frac{\pi}{2}\eta\right]_0^1 = 1 - \frac{2}{\pi} = 0.363$