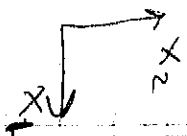


→ U_0

NS-Example

Eric, here is example I did



→ $-V_0$

Viscous flow over flat plate solution

Problem statement:

$$u_1(x_2=0) = 0$$

$$u_1(x_2=\infty) = U_0$$

$$u_2(x_2=0) = -V_0$$

$$P = \text{const}$$

fully developed

(infinite in x_1)

incomp, steady \rightarrow fully developed equations

$$\text{mass} \quad \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} = 0$$

\rightarrow steady \rightarrow fully developed

$$\text{x-mom} \quad \frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x_1} + u_2 \frac{\partial u_1}{\partial x_2} = -\frac{1}{\rho} \frac{\partial p}{\partial x_1} + \nu \frac{\partial^2 u_1}{\partial x_2^2} + \nu \frac{\partial^3 u_1}{\partial x_1^2}$$

\rightarrow pressure grad

\rightarrow fully dev

(A) $\Rightarrow u_2 = C$ evaluate at $x_2=0$ gives $u_2 = -V_0$ everyw

Then (B) is $-V_0 \frac{\partial u_1}{\partial x_1} = \nu \frac{d^2 u_1}{dx_2^2}$

integ $\Rightarrow \frac{du_1}{dx_2} + \frac{\nu}{V_0} u_1 = C$

1st order inhomog equation:

Particular solution, $u_1 = \frac{C V_0}{\nu}$

homogeneous solution:

$$u_1 = B e^{-\frac{\gamma}{V_0} x_2}$$

So general solution is

$$u_1 = B e^{-\frac{\gamma}{V_0} x_2} + C$$

$$u_1(x_2 = \infty) = V_0 \Rightarrow C = V_0$$

$$u_1(x_2 = 0) = 0 \Rightarrow B = -V_0$$

$$\text{So } u_1 = V_0 (1 - e^{-\frac{\gamma}{V_0} x_2})$$

interesting to look at cases $\gamma = 0$

$$V_0 = 0$$

$$V_0 = \infty$$

$$\gamma = \infty$$