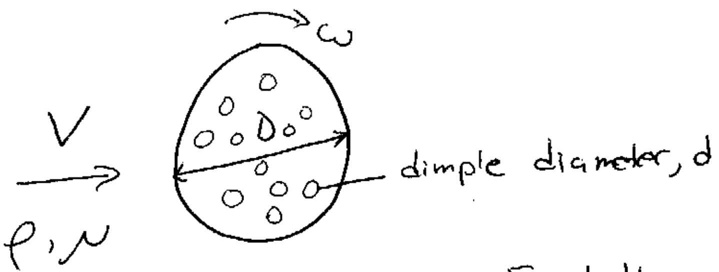
Homework # 3

Key

1.



Find the dimensionless torque, T & other Π groups

$T = (\rho, \mu, V, d, D, \omega)$

$n = 7$ dimensional parameters

note angles are dimensionless

T	ρ	μ	V	d	D	ω
$\frac{ML^2}{T^2}$	$\frac{M}{L^3}$	$\frac{M}{LT}$	$\frac{L}{T}$	L	L	$\frac{1}{T}$

- Choose primary dimension system - $MLT \Rightarrow m = 3$

$\therefore r \leq 3$

- Dimensional matrix

	T	ρ	μ	V	d	D	ω
M	1	1	1	0	0	0	0
L	2	-3	-1	1	1	1	0
T	-2	0	-1	-1	0	0	-1

$$\begin{vmatrix} 1 & 1 & 0 \\ -3 & -1 & 1 \\ 0 & -1 & -1 \end{vmatrix} = (1)(1+1) - (1)(3-0) + 0(\quad) \neq 0$$

$\therefore$ rank $= r = 3$

$\therefore n - r = 4$ Π groups

* Select repeating variables: $\rho V D$

$$\Pi_1 = \rho^a V^b D^c T = M^0 L^0 T^0$$

$$\left(\frac{M}{L^3}\right)^a \left(\frac{L}{T}\right)^b L^c \frac{ML^2}{T^2} = M^0 L^0 T^0$$

M: $a + 1 = 0 \Rightarrow a = -1$

L: $-3a + b + c + 2 = 0 \Rightarrow c = -3$

T: $-b - 2 = 0 \quad b = -2$

$$\Rightarrow \boxed{\Pi_1 = \frac{T}{\rho V^2 D^3}}$$

1. Continued

$$\Pi_2 = \mu \rho^a V^b D^c = M^0 L^0 T^0$$

$$\frac{M}{LT}\left(\frac{M}{L^3}\right)^a\left(\frac{L}{T}\right)^b L^c$$

M: $1 + a = 0 \quad a = -1$

L: $-1 - 3a + b + c = 0 \Rightarrow c = 1 + 3(-1) + 1 = -1$

T: $-1 - b = 0 \Rightarrow b = -1$

$$\therefore \quad \Pi_2 = \frac{\mu}{\rho V D} = \frac{1}{Re_D}$$

from inspection:

$$\Pi_3 = \frac{d}{D}$$

$$\Pi_4 = \omega \rho^a V^b D^c = M^0 L^0 T^0$$

$$\left(\frac{1}{T}\right)\left(\frac{M}{L^3}\right)^a\left(\frac{L}{T}\right)^b L^c$$

M: $a = 0$

L: $-3a + b + c = 0 \quad c = 1$

T: $-1 - b = 0 \Rightarrow b = -1$

$$\Rightarrow \boxed{\Pi_4 = \frac{\omega D}{V}}$$

(2) $\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$

A B C

(a) If $A \sim 0$ what is non-dimensional pressure :

$$\frac{\partial p}{\partial x} = \nu\frac{\partial^2 u}{\partial y^2}$$

$$u_* = \frac{u}{U} \quad x_* = \frac{x}{L}$$

$$p^* = \frac{p}{p_s}$$

$$\frac{1}{L}\frac{\partial p}{\partial x_*} = \nu\frac{U}{L^2}\frac{\partial^2 u_*}{\partial y_*^2}$$

$$\frac{\partial p}{\partial x_*}\frac{L}{\nu U} = \frac{\partial^2 u_*}{\partial y_*^2}$$

Hence, $p^* = \frac{p}{p_s} \Rightarrow p_s = \frac{\nu U}{L}$

(b) If $C \sim 0$

$$\rho\left(u\frac{\partial u}{\partial x}\right) = -\frac{\partial p}{\partial x}$$

$$\rho\frac{U^2}{L}\left(u_*\frac{\partial u_*}{\partial x_*}\right) = -\frac{1}{L}\frac{\partial p}{\partial x_*}$$

$$u_*\frac{\partial u_*}{\partial x_*} = -\frac{\partial p}{\partial x_*}\frac{1}{\rho U^2} \Rightarrow p_s = \rho U^2$$

3.

Exercise 8.1

Given $$P = f(d, U, \omega, c, \rho, \mu)$$

Dimension of power is $[P] = [W]/T = FL/T = (MLT^{-2})L/T = ML^2/T^3$.
The dimensional matrix is

	P	d	U	ω	c	ρ	μ
M	1	0	0	0	0	1	1
L	2	1	1	0	1	-3	-1
T	-3	0	-1	-1	-1	0	-1

We find

$$\begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ -3 & 0 & -1 \end{vmatrix} = 1(-1 - 0) = -1 \neq 0$$

Rank = 3, variables = 7. $\therefore$ # of nondimensional numbers = 4.
We choose U, d and ρ as repeating variables.

Let $\Pi_1 = U^a d^b \rho^c P$

$$\therefore M^0 L^0 T^0 = (LT^{-1})^a (L)^b (ML^{-3})^c ML^2 T^{-3} = M^{1+c} L^{a+b-3c-2} T^{-a-3}$$

Solving we get $a = -3,\ b = -2,\ c = -1$

Thus $$\Pi_1 = P/\rho U^3 d^2$$

Let $\Pi_2 = U^a d^b \rho^c \omega$

$$\therefore M^0 L^0 T^0 = (LT^{-1})^a (L)^b (ML^{-3})^c T^{-1} = M^c L^{a+b-3c} T^{-a-1}$$

Solving we get $a = -1,\ b = 1,\ c = 0$

Thus $$\Pi_2 = d\omega/U$$

Let $\Pi_3 = U^a d^b \rho^c c \longrightarrow \Pi_3 = U/c$

Let $= U^a d^b \rho^c \mu \longrightarrow \Pi_4 = \rho U d/\mu$

Therefore

$$P/\rho U^3 d^2 = f(d\omega/U,\ U/c,\ \rho U d/\mu)$$

At low speeds the power output is likely to depend most strongly on the size and angular speed on the propeller, and viscous effects; so $P/\rho U^3 d^2$ is likely to be determined mostly by $d\omega/U$ and $\rho U d/\mu$. At higher speeds compressibility effects are likely to be more important than viscous effects, so $d\omega/U$ and U/c are most important.

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Exercise 8.2

Given $L_m/L_p = 1/25$

$p_m = 200$ kPa

$T_m = 300\ \text{K} \longrightarrow \rho_m = p_m/RT_m = 2\times10^5/(287)(300) = 2.32\ \text{kg/m}^3$

$U_p = 30$ km/h

$U_m = ?$

$D_m/D_p = ?$

Since the submarine would not operate near the sea surface, gravity is unimportant. Therefore C_D depends only on Re, which should be duplicated in the model test.

Equating $U_m L_m/\nu_m = U_p L_p/\nu_p$ we get

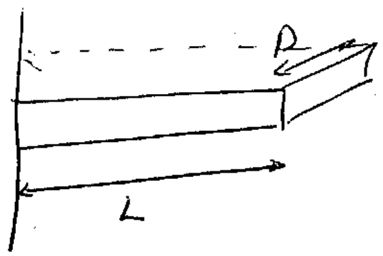
error should be 25

$$U_m = U_p(L_p/L_m)(\nu_m/\nu_p) = 30(1/25)(1.5\times10^{-5}/10^{-6}) = 18\ \text{km/h} = 5\ \text{m/s}$$

Equating C_D, $D_m/\rho_m U_m^2 L_m^2 = D_p/\rho_p U_p^2 L_p^2$

$$D_m/D_p = (\rho_m/\rho_p)(U_m/U_p)^2(L_m/L_p)^2 = (2.32/1000)(18/30)^2(1/25)^2$$
$$= 1.336\times10^{-6}$$

5. (a) $k D^2 \frac{\Delta T}{L^2} - h D \Delta T + \dot{q} D^2 = 0$



As $L \to \infty$

$\frac{1}{L^2} \to 0$

$\therefore k D^2 \frac{\Delta T}{L^2} \to$ negligible

as long as $\frac{D^2}{L^2} \to 0$

(b) $h D \Delta T + \dot{q} D^2 = 0$

$h \cancel{D} \Delta T \sim \dot{q} D^{\cancel{2}}$

$$\boxed{\Delta T \sim \frac{\dot{q} D}{h}}$$

(c) $k \cancel{D^2} \frac{\Delta T}{\delta^2} \sim \dot{q} \cancel{D^2}$

$$\boxed{\delta^2 \sim \frac{k \Delta T}{\dot{q}}}$$