Example 5.1

Use the vorticity transport equation (5.14) and explain the presence of the secondary flow downstream of a bend in a river or creek. Can the typical eroded outer bank and the deposition of stones on the inner bank be explained by such a secondary flow? The velocity profile prior to the bend is as shown in Fig. E5.1. It is known that the flow speeds up on the inside of the bend, as shown at section ②.

Solution. The vorticity at the start of the bend is in the y-direction near the bottom (ω = η j) and in the z-direction on the inner side wall (ω = k), assuming the river to form a rectangular cross-section. The flow on the inside of the bend accelerates (a fact which will be discussed in Chapter 8) resulting, at section ②, in a non-zero ∂u/∂y for the bottom flow and a non-zero ∂u/∂z for the side wall flow. Note that ∂u/∂y = 0 near the bottom and ∂u/∂z = 0 on the sidewall at section ①. The x-component vorticity equation (see Eq. 5.14) is

\[ \frac{D\xi}{Dt} = \xi \frac{\partial u}{\partial x} + \eta \frac{\partial u}{\partial y} + \zeta \frac{\partial u}{\partial z} + \nu \nabla^2 \xi \]

Thus we see that η∂u/∂y from the bottom flow results in a positive Dξ/Dt and hence a positive ξ at section ②. Likewise, ξ∂u/∂z from the sidewall results in a positive ξ. The sense of ξ is to cause a net flow from the outer bank to the inner bank near the bottom of the stream. (If your right thumb points in the direction of the vorticity your curled fingers will indicate the vortical motion.) This is the agent which transports stones to the inner bank. The ξ-vorticity component also causes the surface flow to move toward the outer bank after causing erosion near the stream surface. (This outward flow may be observed by dropping a leaf on the water surface upstream of the bend.)

Example 5.2

The second circulation-producing term in Eq. 5.21, \( -\oint_C (dp/\rho) \), is nonzero if the density is a variable and is not related uniquely to the pressure, that is, if there is a nonbarotropic relationship between \( \rho \) and \( p \). It will be left as an
Example 5.3

At the inlet (see Fig. 5.3.3), explain this observation: 

A relatively thick boundary layer exists at the inlet to a convection. It is observed that the boundary layer at the exit of the connection is smaller than

The fluid is at a higher temperature around the material curve will be developed. Consequently, a circulation around the material curve will be developed. The fluid at the material curve will be elevated upgrate. Since the pressure variation of fluid above fluid, the pressure will be less than the hydrostatic-pressure change from the same initial height would be less than the hydrostatic-pressure change from the same initial height would be less than the hydrostatic-pressure change from the same initial height would be less than the hydrostatic-pressure change from the same initial height.

Solution: A closed contour of fluid particles (a material curve as shown on

Exhibit 5.2.2, explain how the

fluid at a point a stagnation is demonstrated in Fig. 5.2.2, and how the non-orthogonal circulation

A non-orthogonal circulation produced by a fluid flow that is

Note that $\mathbf{d} = \mathbf{0}$ for a barotropic flow, i.e.,

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\[ \phi = 1.18 \text{ kg/m}^3 \text{ at } 25^\circ C \]
\[ p_e = 0 \text{ N/m}^2 \]
\[ p_t = -2000 \text{ N/m}^2 \]

Exercise 5.2

Area uncovered \( = \frac{\pi}{2} \times 0.1 \times 0.71 \text{ m}^2 \)

\[ \therefore A = 0.475 \text{ m}^2 \]

Thus, by subtracting (4) from (3), we get \( z = 0.3 \times 0.491 = 0.1475 \text{ m} \).

(4)

Using (3), where we have used (1) and (2) to replace \( h \) and \( \frac{h}{z} \), the above becomes

\[ z = \frac{81.5}{2}\left( \frac{\pi}{4} r_1^4 \right) - \left( \frac{\pi}{4} r_2^4 \right) \quad \text{or} \quad \frac{V}{4} = \frac{z - r_3^2}{1} \]

Now, volume of air within the tank does not change, so

\[ V = \frac{81.5}{2}\left( \frac{\pi}{4} r_1^4 \right) - \left( \frac{\pi}{4} r_2^4 \right) \]

Subtracting (2) from (1), we get

\[ \frac{\pi}{4} h_1^4 = 81.5 \frac{z}{2} + \frac{h}{z} \]

Angular velocity \( \omega = 40 \text{ rad/s} \)

Chapter 5
\[
\frac{\chi e}{x ne} + \frac{\phi e}{\omega(m_e)} \frac{\chi}{i} + \left(\frac{\chi e}{x ne} \frac{e}{r} \frac{d e}{\chi} \frac{\chi}{i} \frac{1}{n} \right) = \frac{\omega}{i} \Delta
\]

(9)

\[
\begin{align*}
\alpha = & \left(\arctan \frac{x}{r} \frac{\chi}{i} \right) = \frac{\chi e}{x ne} - \left(\frac{d e}{\chi} \frac{\chi}{i} \frac{1}{n} \right) = \frac{\chi}{n} \\
0 & = \frac{\chi e}{x ne} - \frac{\phi e}{\omega(m_e) ne} = \Delta \\
\Delta & = \frac{\chi e}{x ne} - \frac{\phi e}{\omega(m_e) ne} = \frac{\chi}{n}
\end{align*}
\]

(a) From Appendix B the vorticity components are given:

\[
\begin{align*}
0 & = \frac{\chi}{n} \\
\max & = \frac{\phi}{n} \\
0 & = \frac{\chi}{n}
\end{align*}
\]

Exercise 5.3

Time required = \(15 - 0.6 = 14.4\) s

\[\begin{align*}
\rho & = 30.0 \\
\text{Since circulation outside core is constant, } \int_0^r \rho \, d\rho = \int_r^{2r} \rho \, d\rho
\end{align*}\]

So

\[\int_0^r \rho \, d\rho = \frac{2.90}{m/r} \int_0^{2r} \rho \, d\rho = \frac{2(2000)}{1.18} \int_0^{2r} \rho \, d\rho = \frac{2}{1.18} \int_0^{2(2000)} \rho \, d\rho
\]

(b) Apply Bernoulli equation between points 2 and 1:

\[\begin{align*}
\frac{3485}{2} \frac{m}{s} & = \frac{2}{1.18} \int_0^{2(2000)} \rho \, d\rho = \frac{1}{2} \rho \, d\rho = \frac{1}{2} \left(\int_0^2 \rho \, d\rho + \int_0^r \rho \, d\rho\right)
\end{align*}\]

We get

\[\begin{align*}
\nu & = 25 \frac{m}{s}
\end{align*}\]

(a) Applying Bernoulli equation between intensity and edge of core,
Therefore
\[ \frac{dy}{dt} = \pm \sqrt{\lambda} \]
\[ \frac{dx}{dt} = \pm \sqrt{\lambda} \]

Path lines are given by
\[ y = \frac{x^2 + x}{2} \pm \sqrt{\frac{x^2 + x}{2}} \pm \sqrt{\lambda} \]
\[ x = \frac{y^2 + y}{2} \pm \sqrt{\frac{y^2 + y}{2}} \pm \sqrt{\lambda} \]

Three image vortices are
The components of the net velocity at point \((x', y')\) due to the

\[ \frac{x'}{x} \pm \frac{y'}{y} = \lambda \]
\[ \frac{x'}{x} \pm \frac{y'}{y} = \lambda \]
\[ \frac{x'}{x} \pm \frac{y'}{y} = \lambda \]

Exercise 5.9
Some up to the only non-zero component of velocity, changing the sign of \(\tau\)-coordinate and the sign of \(\tau\)-coordinate. This is the net plane of

\[ \lambda_x = \text{constant} \]

Integrating...
(c) Vortex lines are given by
\[ 0 = \frac{2\pi}{e} + \frac{2\pi}{e} \]
\[
\begin{align*}
(1) \quad \nabla \cdot \mathbf{F} &= \mathbf{n} \cdot (\nabla \times \mathbf{u}) \mathbf{u} - \mathbf{u} \times (\mathbf{u} \times \mathbf{n}) \\
\mathbf{k} \cdot (\nabla \times \mathbf{u}) \mathbf{u} - \mathbf{u} \times (\mathbf{u} \times \mathbf{n}) &= \mathbf{k} \cdot \mathbf{n} \times (\mathbf{u} \times \mathbf{n}) - \mathbf{u} \times (\mathbf{u} \times \mathbf{n})
\end{align*}
\]

The extra term here is the Coriolis force. Taking its dot product with element \(dx\) parallel to a circuit \(C\), we get

\[
0 = \left( \frac{d}{dt} \right) \frac{\mathbf{D}}{\mathbf{n}}
\]

Conservative body forces, we showed that

under barotropic and inviscid conditions, and in the presence of

In section 5.4, we took the dot product of each term with \(dx\).

\[
\nabla \cdot \mathbf{u} = \mathbf{n} \times \mathbf{u} - \frac{\partial}{\partial t} \frac{\mathbf{D}}{\mathbf{n}} = \frac{1}{\mathbf{n}} \mathbf{D}
\]

Equation of motion in rotating coordinates is

Exercise 5.5.6

constant.

Initial coordinates of the vortex, thus \(F(t)\) has to be a

Initial condition gives \(I = \mathbf{v} + \mathbf{z}\) where \((x, y, z)\) are the

Integration gives

\[
\frac{x}{dx} = \frac{\mathbf{v}}{\mathbf{p}}
\]

dividing we get

\[
\left[ \frac{(z^2 + x^2) \mathbf{v}}{z} \right] \mathbf{n} = \frac{\mathbf{p}}{\mathbf{p}}
\]

\[
\left[ (z^2 + x^2) \mathbf{v} \right] \mathbf{n} = \frac{\mathbf{p}}{\mathbf{p}}
\]