16-50. At the instant shown the boomerang has an angular velocity \( \omega = 4 \text{ rad/s} \), and its mass center \( G \) has a velocity \( v_G = 6 \text{ in./s} \). Determine the velocity of point \( B \) at this instant.

\[
v_B = v_G + v_{B/G}
\]

\[
v_B = 6 + [4(1.5/\sin 45^\circ) \times 30^\circ/ \sin 45^\circ] = 8.4852 \text{ in./s}
\]

\[
(\uparrow \uparrow) (v_B)_x = 6 \cos 30^\circ + 0 = 5.196 \text{ in./s}
\]

\[
(+ \uparrow) (v_B)_y = 6 \sin 30^\circ + 8.4852 = 11.485 \text{ in./s}
\]

\[
v_B = \sqrt{(5.196)^2 + (11.485)^2} = 12.6 \text{ in./s}
\]

\[
\theta = \tan^{-1} \frac{11.485}{5.196} = 65.7^\circ
\]

Also:

\[
v_B = v_G + \omega \times r_{B/G}
\]

\[
(v_B)_x = (-6 \cos 30^\circ + 6 \sin 30^\circ) + (4k) \times (1.5/\sin 45^\circ)\i
\]

\[
(v_B)_x = -6 \cos 30^\circ = -5.196 \text{ in./s}
\]

\[
(v_B)_y = 6 \sin 30^\circ + 8.4853 = 11.485 \text{ in./s}
\]

\[
v_B = \sqrt{(5.196)^2 + (11.485)^2} = 12.6 \text{ in./s}
\]

\[
\theta = \tan^{-1} \frac{11.485}{5.196} = 65.7^\circ
\]
16-53. The pinion gear rolls on the gear racks. If \( B \) is moving to the right at 8 ft/s and \( C \) is moving to the left at 4 ft/s, determine the angular velocity of the pinion gear and the velocity of its center \( A \).

\[
v_C = v_B + v_{CB}
\]

\[
-4 = 8 - 0.6(\omega)
\]

\[
\omega = 20 \text{ rad/s} \quad \text{Ans}
\]

\[
v_A = v_B + v_{A/B}
\]

\[
-4i = 8i + (\omega k) \times (0.6j)
\]

\[
\omega = 20 \text{ rad/s} \quad \text{Ans}
\]

\[
v_A = 8 - 20(0.3)
\]

\[
v_A = 2 \text{ ft/s} \quad \text{Ans}
\]

Also:

\[
v_C = v_B + \omega \times r_{CB}
\]

\[
v_A = 8i + 20k \times (0.3j)
\]

\[
v_A = 2 \text{ ft/s} \quad \text{Ans}
\]
16-62. At the instant shown, the truck is traveling to the right at 8 m/s. If the spool does not slip at B, determine its angular velocity so that its mass center $G$ appears to an observer on the ground to remain stationary.

\[ v_G = v_B + v_{G/B} \]

\[ 0 = 8 + 1.5\omega \]

\[ \omega = \frac{8}{1.5} = 5.33 \text{ rad/s} \quad \text{Ans} \]

Also:

\[ v_G = v_B + \omega \times r_{G/B} \]

\[ 0i = 8i + (\omega k) \times (1.5j) \]

\[ 0 = 8 - 1.5\omega \]

\[ \omega = \frac{8}{1.5} = 5.33 \text{ rad/s} \quad \text{Ans} \]
*16-88. At the instant shown, the disk is rotating at \( \omega = 4 \text{ rad/s} \). Determine the velocities of points A, B, and C.

The instantaneous center is located at point A. Hence, \( v_A = 0 \)

\[
\begin{align*}
r_{C/IC} &= \sqrt{0.15^2 + 0.15^2} = 0.2121 \text{ m} \quad &r_{B/IC} &= 0.3 \text{ m} \\
v_B &= \omega r_{B/IC} = 4(0.3) = 1.2 \text{ m/s} \\
v_C &= \omega r_{C/IC} = 4(0.2121) = 0.849 \text{ m/s} \quad &\angle &= 45^\circ
\end{align*}
\]
16-90. If link $CD$ has an angular velocity of $\omega_{CD} = 6$ rad/s, determine the velocity of point $E$ on link $BC$ and the angular velocity of link $AB$ at the instant shown.

$$ v_C = \omega_{CD} r_{CD} = (6)(0.6) = 3.60 \text{ m/s} $$

$$ \omega_{BC} = \frac{v_C}{r_{CI/C}} = \frac{3.60}{0.6 \tan30^\circ} = 10.39 \text{ rad/s} $$

$$ v_B = \omega_{BC} r_{BI/C} = (10.39) \left( \frac{0.6}{\cos30^\circ} \right) = 7.20 \text{ m/s} $$

$$ \omega_{AB} = \frac{v_B}{r_{AB}} = \frac{7.20}{\left( \frac{0.6}{\sin30^\circ} \right)} = 6 \text{ rad/s} \quad \text{Ans} $$

$$ v_E = \omega_{BC} r_{EI/C} = 10.39 \sqrt{(0.6 \tan30^\circ)^2 + (0.3)^2} = 4.76 \text{ m/s} \quad \text{Ans} $$

$$ \theta = \tan^{-1} \left( \frac{0.3}{0.6 \tan30^\circ} \right) = 40.9^\circ \quad \text{Ans} $$
16-93. As the car travels forward at 80 ft/s on a wet road, due to slipping, the rear wheels have an angular velocity $\omega = 100$ rad/s. Determine the speeds of points $A$, $B$, and $C$ caused by the motion.

$r = \frac{80}{100} = 0.8$ ft

$v_A = 0.6(100) = 60.0$ ft/s $\rightarrow$ Ans

$v_C = 2.2(100) = 220$ ft/s $\leftarrow$ Ans

$v_B = 1.612(100) = 161$ ft/s $60.3^\circ$ Ans
16-105. At a given instant the bottom $A$ of the ladder has an acceleration $a_A = 4 \text{ ft/s}^2$ and velocity $v_A = 6 \text{ ft/s}$, both acting to the left. Determine the acceleration of the top of the ladder, $B$, and the ladder's angular acceleration at this same instant.

$$\omega = \frac{6}{8} = 0.75 \text{ rad/s}$$

$$a_y = a_A + (a_{y/A})_x + (a_{y/A})_z$$

$$a_y = 4 + (0.75)^2(16) + \alpha(16)$$

$$0 = 4 + (0.75)^2(16) \cos 30° - \alpha(16) \sin 30°$$

$$a_y = 0 + (0.75)^2(16) \sin 30° + \alpha(16) \cos 30°$$

Solving,

$$\alpha = 1.47 \text{ rad/s}^2 \quad \text{Ans}$$

$$a_y = 24.9 \text{ ft/s}^2 \quad \text{Ans}$$

Also:

$$a_y = a_A + \alpha \times r_{B/A} - \omega^2 r_{B/A}$$

$$-a_y \mathbf{j} = -4i + (\alpha \mathbf{k}) \times (16 \cos 30° \mathbf{i} + 16 \sin 30° \mathbf{j}) - (0.75)^2(16 \cos 30° \mathbf{i} + 16 \sin 30° \mathbf{j})$$

$$0 = -4 - 8\alpha - 7.794$$

$$-a_y = 13.856\alpha - 4.5$$

$$\alpha = 1.47 \text{ rad/s}^2 \quad \text{Ans}$$

$$a_y = 24.9 \text{ ft/s}^2 \quad \text{Ans}$$
16-110. At a given instant the wheel is rotating with the angular motions shown. Determine the acceleration of the collar at A at this instant.

See Prob. 16-59,

\[ \omega = 4.16 \text{ rad/s} \]

\[ a_A = a_B + a_{A/B} \]

\[ a_A = 2.4 + 9.6 + (4.16^2)(0.5) + \alpha(0.5) \]

\[ \left( \begin{array}{c} \omega \, 60^\circ \end{array} \right) \]

\[ a_A = 2.4 \cos 60^\circ + 9.6 \cos 30^\circ - 8.65 \cos 60^\circ - \alpha(0.5) \sin 60^\circ \]

\[ \left( \begin{array}{c} + \alpha \end{array} \right) \]

\[ 0 = 2.4 \sin 60^\circ - 9.6 \sin 30^\circ - 8.65 \sin 60^\circ + \alpha(0.5) \cos 60^\circ \]

\[ \alpha = 40.8 \text{ rad/s}^2 \]

\[ a_A = 12.5 \text{ m/s}^2 \quad \text{Ans} \]

Also:

\[ a_A = a_B + \alpha \times r_{A/B} - \omega^2 r_{A/B} \]

\[ a_A = (8^2)(0.15)(\cos 30^\circ)i - (8^2)(0.15)(\sin 30^\circ)j + (16)(0.15)(\sin 30^\circ)i + (16)(0.15)(\cos 30^\circ)j \]

\[ + (\alpha k) \times (0.5 \cos 60^\circ i + 0.5 \sin 60^\circ j) - (4.16)^2(0.5 \cos 60^\circ i + 0.5 \sin 60^\circ j) \]

\[ a_A = 8.314 + 1.200 - 0.433 \alpha - 4.326 \]

\[ 0 = -4.800 + 2.0785 + 0.25 \alpha - 7.4935 \]

\[ \alpha = 40.8 \text{ rad/s}^2 \]

\[ a_A = 12.5 \text{ m/s}^2 \quad \text{Ans} \]
16-114. The disk is moving to the left such that it has an angular acceleration $\alpha = 8 \text{ rad/s}^2$ and angular velocity $\omega = 3 \text{ rad/s}$ at the instant shown. If it does not slip at $A$, determine the acceleration of point $D$.

$$a_C = 0.5(8) = 4 \text{ m/s}^2$$

$$a_D = a_C + a_{DC}$$

$$a_D = \begin{bmatrix} 4 \\ (3)^2(0.5) \\ 4(\cos 45^\circ) \\ 4(\sin 45^\circ) \end{bmatrix} + \begin{bmatrix} 8(0.5) \\ 4(\cos 45^\circ) \\ 4(\sin 45^\circ) \end{bmatrix}$$

$$a_D = \begin{bmatrix} (4 + (3)^2)(0.5) \\ 8(0.5) \\ 4(\cos 45^\circ) \\ 4(\sin 45^\circ) \end{bmatrix}$$

$$\theta = \tan^{-1}\left(\frac{0.3536}{10.01}\right) = 2.02^\circ \quad \text{Ans}$$

$$a_D = \sqrt{(-10.01)^2 + (-0.3536)^2} = 10.0 \text{ m/s}^2 \quad \checkmark \quad \text{Ans}$$

Also,

$$a_D = a_C + \alpha x r_{DC} - \omega^2 r_{DC}$$

$$(a_D)_x = -4 - 8(0.5) x (0.5\cos 45^\circ) - (3)^2(0.5\cos 45^\circ) = -10.01 \text{ m/s}^2$$

$$(+ \uparrow) \quad (a_D)_y = +8(0.5\sin 45^\circ) - (3)^2(0.5\sin 45^\circ) = -0.3536 \text{ m/s}^2$$

$$\theta = \tan^{-1}\left(\frac{0.3536}{10.01}\right) = 2.02^\circ \quad \checkmark \quad \text{Ans}$$

$$a_D = \sqrt{(-10.01)^2 + (-0.3536)^2} = 10.0 \text{ m/s}^2 \quad \checkmark \quad \text{Ans}$$
16-131. Block $A$, which is attached to a cord, moves along the slot of a horizontal forked rod. At the instant shown, the cord is pulled down through the hole at $O$ with an acceleration of $4 \text{ m/s}^2$ and its velocity is $2 \text{ m/s}$. Determine the acceleration of the block at this instant. The rod rotates about $O$ with a constant angular velocity $\omega = 4 \text{ rad/s}$.

**Motion of moving reference.**

$v_O = 0$

$a_O = 0$

$\Omega = 4k$

$\dot{\Omega} = 0$

**Motion of $A$ with respect to moving reference.**

$r_{A/O} = 0.1i$

$v_{A/O} = -2i$

$a_{A/O} = -4i$

Thus,

$$a_A = a_O + \Omega \times r_{A/O} + \Omega \times (\Omega \times r_{A/O}) + 2\Omega \times (v_{A/O})_{xyz} + (a_{A/O})_{xyz}$$

$$= 0 + 0 + (4k) \times (4k \times 0.1i) + 2(4k \times (-2i)) - 4i$$

$$a_A = \{-5.60i - 16\} \text{ m/s}^2 \quad \text{Ans}$$
Block $B$ moves along the slot in the platform with a constant speed of 2 ft/s, measured relative to the platform in the direction shown. If the platform is rotating at a constant rate of $\omega = 5$ rad/s, determine the velocity and acceleration of the block at the instant $\theta = 60^\circ$.

\[
r_{B/0} = \frac{2}{\tan 60^\circ} i + 2j = (1.55i + 2j) \text{ ft}
\]

\[
v_B = v_0 + \Omega \times r_{B/0} + (v_{B/0})_{xyz}
\]

\[
v_B = 0 + 5k \times (1.155i + 2j) - 2i
\]

\[
v_B = (-12.0i + 5.77j) \text{ ft/s} \quad \text{Ans}
\]

\[
a_B = a_0 + \Omega \times r_{B/0} + \Omega \times (\Omega \times r_{B/0}) + 2\Omega \times (v_{B/0})_{xyz} + (a_{B/0})_{xyz}
\]

\[
a_B = 0 + 0 + 5k \times [(5k) \times (1.155i + 2j)] + 2(5k) \times (-2i) + 0
\]

\[
a_B = 0 + 0 - 28.87i - 50j - 20j
\]

\[
a_B = (-28.9i - 70.0j) \text{ ft/s}^2 \quad \text{Ans}
\]
16-139. Rod $AB$ rotates counterclockwise with a constant angular velocity $\omega = 3 \text{ rad/s}$. Determine the velocity and acceleration of point $C$ located on the double collar when $\theta = 45^\circ$. The collar consists of two pin-connected slider blocks which are constrained to move along the circular path and the rod $AB$.

$$r_{CIA} = \{0.400i + 0.400j\}$$

$$v_C = -v_Ci$$

$$v_C = v_A + \Omega \times r_{CIA} + (v_{CIA})_{xyz}$$

$$-v_Ci = 0 + (3k) \times (0.400i + 0.400j) + (v_{CIA})\cos 45^\circ i + (v_{CIA})\sin 45^\circ j$$

$$-v_Ci = 0 - 1.20i + 1.20j + 0.707v_{CIA}i + 0.707v_{CIA}j$$

$$-v_C = -1.20 + 0.707v_{CIA}$$

$$0 = 1.20 + 0.707v_{CIA}$$

$$v_C = 2.40 \text{ m/s} \quad \text{Ans}$$

$$v_{CIA} = -1.697 \text{ m/s}$$

$$a_C = a_A + \Omega \times r_{CIA} + \Omega \times (\Omega \times r_{CIA}) + 2\Omega \times (v_{CIA})_{xyz} + (a_{CIA})_{xyz}$$

$$-(a_{c})i - \frac{(2.40)^2}{0.4}j = 0 + 0 + 3k \times [3k \times (0.4i + 0.4j)] + 2(3k) \times [0.707(-1.697)i + 0.707(-1.697)j]$$

$$-(a_{c})i - 14.40j = 0 + 0 - 3.60i - 3.60j + 7.20i - 7.20j + 0.707a_{CIA}i + 0.707a_{CIA}j$$

$$-(a_{c})i = -3.60 + 7.20 + 0.707a_{CIA}$$

$$-14.40 = -3.60 - 7.20 + 0.707a_{CIA}$$

$$a_{CIA} = -5.09 \text{ m/s}^2$$

$$a_C = a_{CIA} = -5.09 \text{ m/s}^2$$

Thus,

$$a_C = (a_C)_n = \frac{(2.40)^2}{0.4} = 14.4 \text{ m/s}^2$$

$$a_C = \{-14.4j\} \text{ m/s}^2 \quad \text{Ans}$$