The sports car, having a mass of 1700 kg, is traveling along a 20° banked track which has a radius of curvature of \( \rho = 100 \) m. If the coefficient of static friction between the tires and the road is \( \mu_s = 0.2 \), determine the maximum constant speed at which the car can travel without sliding up the slope. Neglect the size of the car.

\[
\sum F = 0: \quad N \cos 20° - 0.2N \sin 20° - 1700(9.81) = 0
\]

\[ N = 19140.6 \text{ N} \]

\[
\sum F = ma: \quad 19140.6 \sin 20° + 0.2(19140.6) \cos 20° = 1700 \left( \frac{v_{\text{max}}}{100} \right)
\]

\[ v_{\text{max}} = 24.4 \text{ m/s} \]

Ans
13-54. Using the data in Prob. 13-53, determine the minimum speed at which the car can travel around the track without sliding down the slope.

\[ + \uparrow \Sigma F_b = 0; \quad N \cos 20^\circ + 0.2N \sin 20^\circ - 1700(9.81) = 0 \]

\[ N = 16543.1 \text{ N} \]

\[ i - \Sigma F_n = ma_n; \quad 16543.1 \sin 20^\circ - 0.2(16543.1) \cos 20^\circ = 1700 \left( \frac{v_{\text{min}}^2}{100} \right) \]

\[ v_{\text{min}} = 12.2 \text{ m/s} \quad \text{Ans} \]
13-60. At the instant $\theta = 60^\circ$, the boy's center of mass $G$ is momentarily at rest. Determine his speed and the tension in each of the two supporting cords of the swing when $\theta = 90^\circ$. The boy has a weight of 60 lb. Neglect his size and the mass of the seat and cords.

\[ \Sigma F = ma; \quad 60 \cos \theta = \frac{60}{32.2} a, \quad a = 32.2 \cos \theta \]

\[ \Sigma F_n = ma; \quad 2T - 60 \sin \theta = \frac{60}{32.2} \left( \frac{v^2}{10} \right) \]  

\[ v \, dv = a \, ds \quad \text{however} \quad ds = 10 \, d\theta \]

\[ \int_0^\circ v \, dv = \int_{60^\circ}^{90^\circ} 322 \cos \theta \, d\theta \]

\[ v = 9.289 \text{ ft/s} \]

From Eq. [1] 
\[ 2T - 60 \sin 90^\circ = \frac{60}{32.2} \left( \frac{9.289^2}{10} \right) \]

\[ T = 38.0 \text{ lb} \]
A particle, having a mass of 1.5 kg, moves along a path defined by the equations \( r = (4 + 3t) \) m, \( \theta = (t^2 + 2) \) rad, and \( z = (6 - t^2) \) m, where \( t \) is in seconds. Determine the \( r \), \( \theta \), and \( z \) components of force which the path exerts on the particle when \( t = 2 \) s.

\[
\begin{align*}
\dot{r} &= 3 \text{ m/s} & \ddot{r} &= 0 \\
\dot{\theta} &= 4 \text{ rad/s} & \ddot{\theta} &= 2 \text{ rad/s}^2 \\
\dot{z} &= -3t^2 & \ddot{z} &= -6 \text{ m/s}^2
\end{align*}
\]

\[
\begin{align*}
s &= -r \dot{\theta}^2 = 0 - 10(4)^2 = -160 \text{ m/s}^2 \\
\ddot{s} &= r \ddot{\theta} + 2r \dot{\theta} = 10(2) + 2(3)(4) = 44 \text{ m/s}^2 \\
\dot{z} &= -12 \text{ m/s}^2 \\
\end{align*}
\]

\[\Sigma F_r = ma_r; \quad F_r = 1.5(-160) = -240 \text{ N} \quad \text{Ans}\]

\[\Sigma F_\theta = ma_\theta; \quad F_\theta = 1.5(44) = 66 \text{ N} \quad \text{Ans}\]

\[\Sigma F_z = ma_z; \quad F_z = 1.5(9.81) = 1.5(-12) \quad F_z = -3.28 \text{ N} \quad \text{Ans}\]
*13-88. The boy of mass 40 kg is sliding down the spiral slide at a constant speed such that his position, measured from the top of the chute, has components \( r = 1.5 \text{ m} \), \( \theta = (0.7t) \text{ rad} \), and \( z = (-0.5t) \text{ m} \), where \( t \) is in seconds. Determine the components of force \( F_r \), \( F_\theta \), and \( F_z \) which the slide exerts on him at the instant \( t = 2 \text{ s} \). Neglect the size of the boy.

\[
\begin{align*}
\dot{r} &= r = 1.5 \\
\dot{\theta} &= \dot{\theta} = 0.7 \\
\ddot{z} &= \ddot{z} = 0.5 \\
\end{align*}
\]

\[
\begin{align*}
ar &= \ddot{r} - r(\dot{\theta})^2 = 0 - 1.5(0.7)^2 = -0.735 \\
a_\theta &= r\ddot{\theta} + 2r\dot{\theta}\dot{\theta} = 0 \\
a_z &= \ddot{z} = 0
\end{align*}
\]

\[
\sum F_r = ma_r; \quad F_r = 40(-0.735) = -29.4 \text{ N} \quad \text{Ans}
\]

\[
\sum F_\theta = ma_\theta; \quad F_\theta = 0 \quad \text{Ans}
\]

\[
\sum F_z = ma_z; \quad F_z - 40(9.81) = 0
\]

\[F_z = 392 \text{ N} \quad \text{Ans}\]
340. The 0.5-lb particle is guided along the circular path using the slotted arm guide. If the arm has an angular velocity \( \dot{\theta} = 4 \text{ rad/s} \) and an angular acceleration \( \ddot{\theta} = 8 \text{ rad/s}^2 \) at the instant \( \theta = 30^\circ \), determine the force of the guide on the particle. Motion occurs in the horizontal plane.

\[
r = 2(0.5 \cos \theta) = 1 \cos \theta
\]

\[
\dot{r} = -\sin \theta \dot{\theta}
\]

\[
\ddot{r} = -\cos \theta \dot{\theta}^2 - \sin \theta \ddot{\theta}
\]

At \( \theta = 30^\circ \), \( \dot{\theta} = 4 \text{ rad/s} \) and \( \ddot{\theta} = 8 \text{ rad/s}^2 \)

\( r = 1 \cos 30^\circ = 0.8660 \text{ ft} \)

\( \dot{r} = -\sin 30^\circ (4) = -2 \text{ ft/s} \)

\( \ddot{r} = -\cos 30^\circ (4)^2 - \sin 30^\circ (8) = -17.856 \text{ ft/s}^2 \)

\[
a_r = \ddot{r} - r\dot{\theta}^2 = -17.856 - 0.8660(4)^2 = -31.713 \text{ ft/s}^2
\]

\[
a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.8660(8) + 2(-2)(4) = -9.072 \text{ ft/s}^2
\]

\[
\sum F_r = ma_r; \quad -N \cos 30^\circ = \frac{0.5}{32.2}(-31.713) \quad N = 0.5686 \text{ lb}
\]

\[
+ \sum F_\theta = ma_\theta; \quad F - 0.5686 \sin 30^\circ = \frac{0.5}{32.2}(-9.072)
\]

\( F = 0.143 \text{ lb} \quad \text{Ans} \)
22-1. When a 20-lb weight is suspended from a spring, the spring is stretched a distance of 4 in. Determine the natural frequency and the period of vibration for a 10-lb weight attached to the same spring.

\[ k = \frac{20}{4} = 60 \text{ lb/ft} \]

\[ \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{60}{\frac{10}{32.2}}} = 13.90 \text{ rad/s} \]

\[ \tau = \frac{2\pi}{\omega_n} = 0.452 \text{ s} \quad \text{Ans} \]

\[ f = \frac{1}{\tau} = 2.21 \text{ Hz} \quad \text{Ans} \]
22-5. A 2-lb weight is suspended from a spring having a stiffness \( k = 2 \) lb/in. If the weight is pushed 1 in. upward from its equilibrium position and then released from rest, determine the equation which describes the motion. What is the amplitude and the natural frequency of the vibration?

\[
k = 2(12) = 24 \text{ lb/ft}
\]

\[
\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{24}{\frac{2}{32.2}}} = 19.66 \text{ rad/s}
\]

\[
f = \frac{\omega_n}{2\pi} = 3.13 \text{ Hz}
\]

\[y = -\frac{1}{12}, \quad v = 0 \text{ at } t = 0\]

From Eqs. 22-3 and 22-4,

\[-\frac{1}{12} = 0 + B\]

\[B = -0.0833\]

\[0 = A\omega_n + 0\]

\[A = 0\]

\[C = \sqrt{A^2 + B^2} = 0.0833 \text{ ft} = 1 \text{ in.} \quad \text{Ans}\]

Position equation,

\[y = (0.0833 \cos 19.7t) \text{ ft} \quad \text{Ans}\]
A pendulum has a 0.4-m-long cord and is given a tangential velocity of 0.2 m/s toward the vertical from a position $\theta = 0.3$ rad. Determine the equation which describes the angular motion.

See Example 22-1.

$$\omega_n = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.81}{0.4}} = 4.95$$

$$\theta = A \sin \omega_n t + B \cos \omega_n t$$

$\theta = 0.3$ rad when $t = 0$.

$$0.3 = 0 + B; \quad B = 0.3$$

$$\dot{\theta} = A \omega_n \cos \omega_n t - B \omega_n \sin \omega_n t$$

Since $s = \theta l$, $\dot{s} = \dot{\theta} l$. Hence,

$$-0.2 = \dot{\theta}(0.4), \dot{\theta} = -0.5 \text{ when } t = 0,$$

$$-0.5 = A(4.95); \quad A = -0.101$$

Thus,

$$\theta = -0.101 \sin(4.95t) + 0.3 \cos(4.95t) \quad \text{Ans}$$