16-1. A wheel has an initial clockwise angular velocity of 10 rad/s and a constant angular acceleration of 3 rad/s². Determine the number of revolutions it must undergo to acquire a clockwise angular velocity of 15 rad/s. What time is required?

\[ \omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0) \]

\[ (15)^2 = (10)^2 + 2(3)(0 - 0) \]

\[ \theta = 20.83 \text{ rad} = 20.83 \left( \frac{1}{2\pi} \right) = 3.32 \text{ rev.} \quad \text{Ans} \]

\[ \omega = \omega_0 + \alpha_c t \]

\[ 15 = 10 + 3t \]

\[ t = 1.67 \text{ s} \quad \text{Ans} \]
*16-4. Just after the fan is turned on, the motor gives the blade an angular acceleration \( \alpha = (20e^{-0.6t}) \text{ rad/s}^2 \), where \( t \) is in seconds. Determine the speed of the tip \( P \) of one of the blades when \( t = 3 \) s. How many revolutions has the blade turned in 3 s? When \( t = 0 \) the blade is at rest.

\[
\begin{align*}
\omega &= \alpha \, dt \\
\int_0^\infty \omega \, dt &= \int_0^t 20e^{-0.6t} \, dt \\
\omega &= -\frac{20}{0.6} e^{-0.6t}\bigg|_0^t = 33.3 \left( 1 - e^{-0.6t} \right) \\
\omega &= 27.82 \text{ rad/s}
\end{align*}
\]

\[
\begin{align*}
v_p &= \omega r = 27.82(1.75) = 48.7 \text{ ft/s} \quad \text{Ans}
\end{align*}
\]

\[
\begin{align*}
\theta &= \omega \, dt \\
\int_0^\theta \theta &= \int_0^t 33.3 \left( 1 - e^{-0.6t} \right) \, dt \\
\theta &= 33.3 \left( t + \left( \frac{1}{0.6} \right) e^{-0.6t} \right) \bigg|_0^t = 33.3 \left[ 3 + \left( \frac{1}{0.6} \right) \left( e^{-0.6(3)} - 1 \right) \right] \\
\theta &= 53.63 \text{ rad} = 8.54 \text{ rev} \quad \text{Ans}
\end{align*}
\]
16-8. The pinion gear A on the motor shaft is given a constant angular acceleration \( \alpha = 3 \text{ rad/s}^2 \). If the gears A and B have the dimensions shown, determine the angular velocity and angular displacement of the output shaft C, when \( t = 2 \text{ s} \) starting from rest. The shaft is fixed to B and turns with it.

\[
\begin{align*}
\omega &= \omega_0 + \alpha t \\
\omega_A &= 0 + 3(2) = 6 \text{ rad/s} \\
\theta &= \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \\
\theta_A &= 0 + 0 + \frac{1}{2} (3)(2)^2 \\
\theta_A &= 6 \text{ rad} \\
\omega_A r_A &= \omega_B r_B \\
6(35) &= \omega_B (125) \\
\omega_C &= \omega_B = 1.68 \text{ rad/s} \quad \text{Ans} \\
\theta_A r_A &= \theta_B r_B \\
6(35) &= \theta_B (125) \\
\theta_C &= \theta_B = 1.68 \text{ rad} \quad \text{Ans}
\end{align*}
\]
*16-12. When only two gears are in mesh, the driving gear A and the driven gear B will always turn in opposite directions. In order to get them to turn in the *same direction* an idler gear C is used. In the case shown, determine the angular velocity of gear B when \( t = 5 \) s, if gear A starts from rest and has an angular acceleration of \( \alpha_A = (3t + 2) \text{ rad/s}^2 \), where \( t \) is in seconds.

\[
d\omega = \alpha \, dt
\]

\[
\int_0^{\omega_A} d\omega_A = \int_0^5 (3t + 2) \, dt
\]

\[
\omega_A = 1.5t^2 + 2t \bigg|_{t=5} = 47.5 \text{ rad/s}
\]

\[
(47.5)(50) = \omega_K(50)
\]

\[
\omega_K = 47.5 \text{ rad/s}
\]

\[
\omega_B(75) = 47.5(50)
\]

\[
\omega_B = 31.7 \text{ rad/s} \quad \text{Ans}
\]
16-18. Starting from rest when \( s = 0 \), pulley \( A \) is given an angular acceleration \( \alpha = (6\theta) \text{ rad/s}^2 \), where \( \theta \) is in radians. Determine the speed of block \( B \) when it has risen \( s = 6 \text{ m} \). The pulley has an inner hub \( D \) which is fixed to \( C \) and turns with it.

\[
\alpha_A = 6\theta_A \\
\theta_C = \frac{6}{0.075} = 80 \text{ rad} \\
\theta_A(0.05) = 80(0.15) \\
\theta_A = 240 \text{ rad} \\
\alpha d\theta = \omega d\omega \\
\int_0^{240} 6\theta_A d\theta_A = \int_0^{\omega_A} \omega_A d\omega_A \\
\omega_A = [6(240)^2]^{1/2} = 587.88 \text{ rad/s} \\
(587.88)(0.05) = \omega_C(0.15) \\
\omega_C = 195.96 \\
\nu_B = 195.96(0.075) = 14.7 \text{ m/s} \quad \text{Ans} \\
\text{Also,} \\
\alpha_A = 6\theta_A \\
\text{But } \alpha_A(50) = 150\alpha_C \\
\alpha_A = 3\alpha_C \\
3\alpha_C = 6\theta_A \\
\alpha_C = 2\theta_A \\
\text{But } \theta_A(50) = 150(\theta_C) \\
\theta_A = 3\theta_C \\
\text{Thus, } \alpha_C = 6\theta_C \\
\int_0^{\theta_C} 6\theta_C d\theta_C = \int_0^{\omega_C} \omega_C d\omega_C \\
6\theta_C^2 = \omega_C^2 \\
\omega_C = \frac{6}{0.075} = 80 \text{ rad} \\
\omega_C = \sqrt{6(80)} = 195.96 \\
\nu_B = (195.96)(0.075) = 14.7 \text{ m/s} \quad \text{Ans}
The operation of "reverse" for a three-speed automotive transmission is illustrated schematically in the figure. If the crank shaft $G$ is turning with an angular speed of 60 rad/s, determine the angular speed of the drive shaft $H$. Each of the gears rotates about a fixed axis. Note that gears $A$ and $B$, $C$ and $D$, $E$ and $F$ are in mesh. The radii of each of these gears are reported in the figure.

![Diagram of the transmission system]

$r_A = 90$ mm  
$r_B = r_C = 30$ mm  
$r_D = 50$ mm  
$r_E = 70$ mm  
$r_F = 60$ mm

$60(90) = \omega_{BC}(30)$

$\omega_{BC} = 180$ rad/s

$180(30) = 50(\omega_{DE})$

$\omega_{DE} = 108$ rad/s

$108(70) = (60)(\omega_H)$

$\omega_H = 126$ rad/s  
Ans
16-33. The bar $DC$ rotates uniformly about the shaft at $D$ with a constant angular velocity $\omega$. Determine the velocity and acceleration of the bar $AB$, which is confined by the guides to move vertically.

\[ y = l \sin \theta \]

\[ \dot{y} = v_y = l \cos \theta \dot{\theta} \]

\[ \ddot{y} = a_y = l \left( \cos \theta \ddot{\theta} - \sin \theta \dot{\theta}^2 \right) \]

Here $v_y = v_{AB}$, $a_y = a_{AB}$, and $\dot{\theta} = \omega$, $\ddot{\theta} = \alpha = 0$.

\[ v_{AB} = l \cos \theta (\omega) = \omega \ l \cos \theta \quad \text{Ans} \]

\[ a_{AB} = \left[ \cos \theta(0) - \sin \theta(\omega)^2 \right] = -\omega^2 l \sin \theta \quad \text{Ans} \]
The block moves to the left with a constant velocity \( v_0 \). Determine the angular velocity and angular acceleration of the bar as a function of \( \theta \).

**Position Coordinate Equation**: From the geometry.

\[
x = \frac{a}{\tan \theta} = a \cot \theta
\]  \hspace{1cm} [1]

**Time Derivatives**: Taking the time derivative of Eq. [1], we have

\[
\frac{dx}{dt} = -\alpha \csc^2 \theta \frac{d\theta}{dt}
\]  \hspace{1cm} [2]

Since \( v_0 \) is directed toward negative \( x \), then \( \frac{dx}{dt} = -v_0 \). Also, \( \frac{d\theta}{dt} = \omega \).

From Eq. [2],

\[
-v_0 = -\alpha \csc^2 \theta (\omega)
\]

\[
\omega = \frac{v_0}{\alpha \csc^2 \theta} = \frac{v_0}{a \sin^2 \theta}
\]  \hspace{1cm} \text{Ans}

Here, \( \alpha = \frac{d\omega}{dt} \). Then from the above expression

\[
\alpha = \frac{v_0}{a} (2 \sin \theta \cos \theta) \frac{d\theta}{dt}
\]  \hspace{1cm} [3]

However, \( 2 \sin \theta \cos \theta = \sin 2\theta \) and \( \omega = \frac{d\theta}{dt} = \frac{v_0}{a} \sin^2 \theta \). Substituting these values into Eq. [3] yields

\[
\alpha = \frac{v_0}{a} \sin 2\theta \left( \frac{v_0}{a} \sin^2 \theta \right) = \left( \frac{v_0}{a} \right)^2 \sin 2\theta \sin^2 \theta
\]  \hspace{1cm} \text{Ans}
Disk A rolls without slipping over the surface of the fixed cylinder B. Determine the angular velocity of A if its center C has a speed \( v_C = 5 \text{ m/s} \). How many revolutions will A have made about its center just after link DC completes one revolution?

As shown by the construction, as A rolls through the arc \( s = \theta_A r \), the center of the disk moves through the same distance \( s' = s \).

Hence,

\[ s = \theta_A r \]
\[ s' = \dot{\theta}_A r \]

\[ s = \omega_A (0.15) \]

\[ \omega_A = 33.3 \text{ rad/s} \quad \text{Ans} \]

Link:

\[ s' = 2r \theta_{CD} = s = \theta_A r \]

\[ 2\theta_{CD} = \theta_A \]

Thus, A makes 2 revolutions for each revolution of CD. \quad \text{Ans}