ME3210 MECHATRONICS II
FINAL EXAMINATION
SPRING 2000

NAME ___________________  

SHOW ALL OF YOUR WORK
OPEN BOOK
OPEN NOTES

Correct answers without work shown or wrong methods used will be considered wild guesses or worse and will be marked appropriately.

Answers Must Have Proper Dimensions for Full Credit.

Put the answers in the spaces given where appropriate.
1. (10 pts) Draw the linear graph of the system. Note the sign for positive positions and velocities.

![Linear graph of the system](image)
2. (10 pts) Derive the differential equations for the currents from the following linear graph representation of an electrical circuit. \( n_1 \) and \( n_2 \) are the number of wraps on the transformer coils. The transformer has the following relationship for its voltages:

\[(V_{n1})n_1 = -(V_{n2})n2\]

\[V_{n1} = V(t)\]

\[V_{n2} = L_{i2} I_{i2}\]

\[V_{n3} = L_{c} E_{c}\]

\[V_{n4} = L_{e} E_{e}\]

**Loop 1**: 
\[-V(\pi) + V_R + V_{n1} = 0\]

\[R I_1 + V_{n1} = V(\pi)\]

**Loop 2**: 
\[-V_{n2} - V_c - V_e = 0\]

\[-V_{n2} - \frac{1}{c} \int I_2 dx - L_{i2} \frac{dI_2}{dt} = 0\]

**Loop 3**: 
\[(V_{n1})n_1 = -(V_{n2})n2\]

**Loop 4**: 
\[\frac{I_1}{n_1} = \frac{I_2}{n_2}\]
3. (20 pts) What is the range of stable values of the gain $k$ and what will be the expected frequency at upper value of the gain $K$ at marginal stability? Give your answers to 2 decimal point precision.

$$\frac{C}{R} = \frac{\frac{K(s+5)}{s(s+1)(s+3)}}{1 + \frac{K(s+5)}{s(s+1)(s+3)}} = \frac{K(s+5)}{s(s+1)(s+3) + k(s+5)}$$

Characteristic equation:

$$s(s+1)(s+3) + k(s+5) = 0$$

$$s^3 + 4s^2 + 3s + ks + k^2 = 0$$

Kouth:

$$s^3 + 4s^2 + (3+k)s + 5k = 0$$

$$s^3 + 12s^2 + 32s + 32k = 0$$

$$s^3 + 4s^2 + (3+k)s + 5k = 0$$

$$\frac{4(3+k) - 16}{4} = \frac{12 + 4k - 5k}{4} = \frac{12 - k}{4}$$

So:

$$\frac{12 - k}{4} > 0 \quad \text{or} \quad k < 12$$

And:

$$5k > 0 \quad \text{or} \quad k > 0$$

So:

$$0 < k < 12$$

If $k = 12$,

$$4s^2 + 5(12) = 0$$

$$s^2 = -\frac{5(12)}{4} = -15$$

$$s = \pm \sqrt{15}$$

$$-\omega^3 + 4(1 + 3 + k) \omega + 32k = 0$$

$$-\omega^3 + 4 \omega^2 + (3+k) \omega + 5k = 0$$

Answer:

$$0 \leq k \leq 12$$

$$\omega = \sqrt{15} \text{ rad/sec}$$
Use the following system for the remaining questions.

\[ G(s) = \frac{(s + 5)}{s(s + 1)(s + 3)} \]

\[ H(s) = 1 \]

Root locus plot

1.1 cm = 1 unit
Bode plot of the open loop transfer function for K=1

Bode Diagrams

Frequency (rad/sec)

Magnitude (dB)

Phase (deg)

Bode Diagrams

Frequency (rad/sec)

Magnitude (dB)

Phase (deg)
4. (5 pts) The gain margin = \( \frac{20 \, \text{dB}}{21.9 \, \text{dB}} \)  
(accuracy as read from the plots – Mark the plots)

5. (5 pts) The phase margin = \( \frac{30^\circ}{35.1^\circ} \)  
(accuracy as read from the plots – Mark the plots)
6. (10 pts) For a desired input \( r(t) \) of a ramp of slope 5, what gain will make the steady state error less than or equal to .5?

\[
Q(s) = \frac{5}{s^2} \\
K_v = \lim_{s \to 0} s \left( \frac{K(s+5)}{s(c(s+1)(s+3))} \right) = \frac{K(5)}{3} \\
E_{ss} = \frac{5 \cdot \frac{3}{10}}{\frac{3}{10}} = \frac{1}{2} \\
\text{and } E_{ss} \leq \frac{1}{2} \\
\frac{3}{K} \leq \frac{1}{2} \\
K \geq 6
\]

Gain \( K \geq 6 \)

7. (10 pts) What is error to a unit step disturbance for the gain found in problem 6?

\[
\frac{C(s)}{D(s)} = \frac{\frac{5(s+5)}{s(5s+5)(s+3)}}{1 + \frac{K(c(s+5))}{s(s+1)(s+3)}} = \frac{\frac{5+5}{5(s+1)(s+3) + K(s+5)}}{s+5} \\
D(s) = \frac{1}{s} \Rightarrow K = 6 \\
C(s) \Rightarrow E_{ss} = \lim_{s \to 0} s \left( \frac{1}{s} \right) \left( \frac{\frac{5+5}{5(s+1)(s+3) + 6(s+5)}}{s+5} \right) \\
= \frac{5}{30} = \frac{1}{6}
\]

Steady state error = \( \frac{1}{6} \)
8. (10 pts) What gain will give the closed loop system a time to peak of 3.14 seconds or less with the minimal overshoot to a step input?
\[
T_p \leq \frac{\pi}{\omega_d} \quad \omega_d \geq 1.415
\]
\[
\left(\frac{1.2}{1.1}\right) \left(\frac{1.25}{1.1}\right) \left(\frac{3}{1.1}\right) = 7.22\
\left(\frac{5.15}{1.1}\right)
\]
\[\omega_c = \frac{2470}{1 + 1.6}\]

9. (5 pts) What is the per cent overshoot at the gain of problem 8?
\[\text{angle} = 62^\circ\]
\[
\cos(62^\circ) = 0.49 = 3.73 (\text{at} 1.6)\]
\[
\frac{26.43^\circ}{1.26} = 26.43^\circ\]

10. (5 pts) What is the settling time to 2% of the final value?
\[T_s \approx 1.4\]
\[
\frac{T_s}{\tau} = \frac{4}{1} = 10 \text{ sec}
\]

11. (5 pts) What is the natural frequency?
\[
\frac{1.2}{1.1} = 1.09 \frac{5}{\omega_c} = 1.08 \frac{5}{\omega_c}\]

12. (5 pts) Is the second order approximation appropriate? Why?
\[\text{Old since the new poles are about 4 times away}\]