

# INTEGRAL ESTIMATION IN QUANTUM PHYSICS

by  
Jane Doe

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in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

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# The University of Utah Graduate School

## STATEMENT OF DISSERTATION APPROVAL

The dissertation of Jane Doe  
has been approved by the following supervisory committee members:

<u>Cornelius Lánzos</u> ,	Chair(s)	<u>17 Feb 2016</u> Date Approved
<u>Hans Bethe</u> ,	Member	<u>17 Feb 2016</u> Date Approved
<u>Niels Bohr</u> ,	Member	<u>17 Feb 2016</u> Date Approved
<u>Max Born</u> ,	Member	<u>17 Feb 2016</u> Date Approved
<u>Paul A. M. Dirac</u> ,	Member	<u>17 Feb 2016</u> Date Approved

by Petrus Marcus Aurelius Featherstone-Hough , Chair/Dean of  
the Department/College/School of Mathematics  
and by Alice B. Toklas , Dean of The Graduate School.



For my parents, Alice and Bob.

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## NOTATION AND SYMBOLS

---

$\alpha$	fine-structure (dimensionless) constant, approximately 1/137
$\alpha$	radiation of doubly-ionized helium ions, He <sup>++</sup>
$\beta$	radiation of electrons
$\gamma$	radiation of very high frequency, beyond that of X rays
$\gamma$	Euler's constant, approximately 0.577 215 . . .
$\delta$	stepsize in numerical integration
$\delta(x)$	Dirac's famous function
$\epsilon$	a tiny number, usually in the context of a limit to zero
$\zeta(x)$	the famous Riemann zeta function
. . .	. . .
$\psi(x)$	logarithmic derivative of the gamma function
$\omega$	frequency

---

# CHAPTER 1

## THE FIRST

This is a chapter. Remember that there should *always* be at least of few lines of prose after each sectional heading: failure to do so is a disservice to your readers, and also produces incorrect vertical spacing.

### 1.1 The first section

Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah.

In **Figure 1.1** on the following page, we have a picture, and the  $\LaTeX$  markup to include it looks like this:

```
\begin{figure}[t]
  \centerline{\includegraphics{fig1}}
  \caption{The first figure.}%
  \figlabel{fig1}
\end{figure}
```

We intentionally omitted an extension on the filename, so that this document can be processed with `latex` to get an output `.dvi` file, or with `pdflatex` to get an output `.pdf` file. The first case uses the file `fig1.eps`, and the second uses `fig1.pdf`. The `distill` or `ps2pdf` commands can be used to convert from *Encapulated PostScriptfiles* to *Portable Document Formatfiles*.

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**1.1.3.2.2 The second numbered paragraph** Blah blah blah blah blah blah. Blah blah blah blah blah blah. Blah blah blah blah blah blah. Blah blah blah blah blah blah. Blah blah blah blah blah.

## 1.2 The second section

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In **Figure 1.2**, we have another picture.

Blah blah blah blah. Blah blah blah blah. Blah blah blah blah. Blah blah blah blah. Blah blah blah blah.

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**Figure 1.2.** The second figure.

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In **Table 1.1**, we show the 24-character lowercase Greek alphabet.

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**Table 1.1.** Lowercase Greek letters.

$\alpha$	alpha
$\beta$	beta
$\gamma$	gamma
$\delta$	delta
$\epsilon, \varepsilon$	epsilon
$\zeta$	zeta
$\eta$	eta
$\theta, \vartheta$	theta
$\iota$	iota
$\kappa$	kappa
$\lambda$	lambda
$\mu$	mu
$\nu$	nu
$\xi$	xi
$\omicron$	omicron
$\pi$	pi
$\rho$	rho
$\sigma, \varsigma$	sigma
$\tau$	tau
$\upsilon$	upsilon
$\phi, \varphi$	phi
$\chi$	chi
$\psi$	psi
$\omega$	omega



**Table 1.2.** Uppercase Greek letters. Notice that several have the same letter shapes as Latin letters, and for those, T<sub>E</sub>X does not define macro names. For convenience, we supply our own definitions of these macros: `\Alpha`, `\Beta`, `\Epsilon`, `\Zeta`, `\Eta`, `\Iota`, `\Kappa`, `\Mu`, `\Nu`, `\Omicron`, `\Rho`, `\Tau`, and `\Chi`.

$A$	Alpha
$B$	Beta
$\Gamma$	Gamma
$\Delta$	Delta
$E$	Epsilon
$Z$	Zeta
$H$	Eta
$\Theta$	Theta
$I$	Iota
$K$	Kappa
$\Lambda$	Lambda
$M$	Mu
$N$	Nu
$\Xi$	Xi
$O$	Omicron
$\Pi$	Pi
$P$	Rho
$\Sigma$	Sigma
$T$	Tau
$Y$	Upsilon
$\Phi$	Phi
$X$	Chi
$\Psi$	Psi
$\Omega$	Omega

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## 1.4 Free software packages

The Free Software Foundation offers almost 300 software packages, most easily portable to many different operating systems and CPU platforms. They include at least these:

`a2ps`, `acct`, `acm`, `adns`, `alive`, `anubis`, `apl`, `archimedes`, `aris`, `aspell`, `auctex`, `autoconf-archive`, `autoconf`, `autogen`, `automake`, `avl`, `ballandpaddle`, `barcode`, `bash`, `bayonne`, `bc`, `binutils`, `bison`, `bool`, `bpel2owfn`, `c-graph`, `ccaudio`, `ccd2cue`, `ccrtp`,







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## 1.5 Resizing figures

In **Figure 1.4** through **Figure 1.8** on the following page, we show how graphics files can be rescaled to convenient sizes, with input like this:

```
\begin{figure}[p]
  \centerline{\includegraphics[scale = 0.5]{fig1}}
  \caption{The fourth figure (at 50\% scale).}%
  \figlabel{fig4}
\end{figure}

\begin{figure}[p]
  \centerline{\includegraphics[scale = 0.75]{fig1}}
  \caption{The fifth figure (at 75\% scale).}%
  \figlabel{fig5}
\end{figure}
```



**This is Figure 1**

**Figure 1.4.** The fourth figure (at 50% scale).



**This is Figure 1**

**Figure 1.5.** The fifth figure (at 75% scale).



**This is Figure 1**

**Figure 1.6.** The sixth figure (at native size).



**This is Figure 1**

**Figure 1.7.** The seventh figure (at 125% scale).



**This is Figure 1**

**Figure 1.8.** The eighth figure (at 175% scale).

```

\begin{figure}[p]
  \centerline{\includegraphics{fig1}}
  \caption{The sixth figure (at native size).}%
  \figlabel{fig6}
\end{figure}

\begin{figure}[p]
  \centerline{\includegraphics[scale = 1.25]{fig1}}
  \caption{The seventh figure (at 125\% scale).}%
  \figlabel{fig7}
\end{figure}

\begin{figure}[p]
  \centerline{\includegraphics[scale = 1.75]{fig1}}
  \caption{The eighth figure (at 175\% scale).}%
  \figlabel{fig8}
\end{figure}

```

You can include multiple images, each with its own caption inside a single *unbreakable* figure environment, like this example shown in **Figure 1.9** and **Figure 1.10** on the next page, although you might want to adjust interfigure vertical space with a `\vspace{}` command:

```

\begin{figure}[t]
  \centerline{\includegraphics[scale = 0.5]{fig1}}
  \caption{The fourth figure (at 50\% scale).}%
  \figlabel{fig9}
  \vspace{3ex}
  \centerline{\includegraphics[scale = 0.75]{fig1}}
  \caption{The fifth figure (at 75\% scale).}%
  \figlabel{fig10}
\end{figure}

```

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**This is Figure 1**

**Figure 1.9.** The ninth figure (at 50% scale), boxed with the tenth figure.



**This is Figure 1**

**Figure 1.10.** The tenth figure (at 75% scale), boxed with the ninth figure.

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As a final example in this chapter, **Figure 1.11** on the following page shows how you can use  $\LaTeX$  picture mode for annotating and positioning graphics images prepared outside  $\LaTeX$ . The input that produced that figure looks like this:

```
\begin{figure}[t]
  %% The original image is 216bp wide by 72bp high, but we
  %% rescale it to 150 picture units divided by \unitlength:
  %% 150 / 0.75 = 112.5 mm
  \newcommand {\myfig} {\includegraphics[width = 112.5mm]{fig1}}

  \begin{center}
    %% The \unitlength is chosen to make the complete picture fit
    %% within the page margins

    \setlength{\unitlength}{0.75mm}

    %%%      insert (width,height)(lower-left-x,lower-left-y)
    \begin{picture}(170,70)(10,10)
      %% Place the included image FIRST!
      \put(10,10) {\myfig}

      %% Everything that follows OVERLAYS the original image!

      \graphpaper[10](0,0)(170,70)

      %% Mark the image center and corners by centered bullets
      \newcommand {\thedot} {\makebox (0,0) {$\bullet$}}
      \put( 85, 35) {\thedot}
      \put( 10, 10) {\thedot}
```



blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah blah. Blah blah  
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### **4.3 Summary and conclusions**

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blah blah blah blah blah blah blah blah blah blah blah blah blah blah.





## APPENDIX C

### THE THIRD

This is an appendix.

There are several books [12, 19–21, 23–25, 27–30] listed in our bibliography.

We also reference several journal articles [1, 2, 4, 8–10, 13–18, 22, 31, 32] and three famous doctoral theses of later winners [3, 6, 7] of the Nobel Prize in Physics (1922, 1933, and 1921):

Notice that, even though those citations appeared in  $\text{\LaTeX}$  `\cite{...}` commands with their  $\text{\BibTeX}$  citation labels in reverse alphabetical order, thanks to the `citesort` package, their reference-list numbers have been sorted in numerically ascending order, and then range-reduced.

Mention should also be made of a famous Dutch computer scientist's first publication [5].

Font metrics are an important, albeit low-level, aspect of typesetting. See the *Adobe Systems* manual about that company's procedures [26].

The bibliography at the end of this thesis contains several examples of documents with non-English titles, and their  $\text{\BibTeX}$  entries provide title translations following the practice recommended by the American Mathematical Society and SIAM. Here is a sample entry that shows how to do so:

```
@PhdThesis{Einstein:1905:NBM,  
  author =      "Albert Einstein",  
  title =      "{Eine Neue Bestimmung der Molek{\u}ldimensionen}.  
                ({German}) [{A} new determination of molecular  
                dimensions]",  
  type =      "Inaugural dissertation",  
  school =     "Bern Wyss.",  
  address =    "Bern, Switzerland",  
  year =      "1905",  
  bibdate =   "Fri Dec 17 10:46:57 2004",
```

```

bibsource = "http://www.math.utah.edu/pub/tex/bib/einstein.bib",
note = "Published in \cite{Einstein:1906:NBM}.",
acknowledgement = ack-nhfb,
language = "German",
advisor = "Alfred Kleiner (24 April 1849--3 July 1916)",
URL = "http://en.wikipedia.org/wiki/Alfred_Kleiner",
remark = "Received August 19, 1905 and published February 8,
1906.",
Schilpp-number = "6",
}

```

The note field in that entry refers to another bibliography entry that need not have been directly cited in the document text. Such cross-references are common in  $\text{BIB}\text{T}\text{E}\text{X}$  files, especially for journal articles where there may be later comments and corrigenda that should be mentioned. Embedded `\cite{}` commands ensure that those possibly-important other entries are always included in the reference list when the entry is cited. The last bibliography entry [32] in this thesis has a long note field that tells more about what some may view as the most important paper in mathematics in the last century.

When entries cite other entries that cite other entries that cite other entries that ..., multiple passes of  $\text{L}\text{A}\text{T}\text{E}\text{X}$  and  $\text{BIB}\text{T}\text{E}\text{X}$  are needed to ensure consistency. That is another reason why document compilation should be guided by a `Makefile` or a batch script, rather than expecting the user to remember just how many passes are needed.

$\text{BIB}\text{T}\text{E}\text{X}$  entries are *extensible*, in that arbitrary key/value pairs may be present that are not necessarily recognized by any bibliography style files. The `advisor`, `acknowledgement`, `bibdate`, `bibsource`, `language`, `remark`, and `Schilpp-number` fields are examples, and may be used by other software that processes  $\text{BIB}\text{T}\text{E}\text{X}$  entries, or by humans who read the entries. `DOI` and `URL` fields are currently recognized by only a few styles, but that situation will likely change as publishers demand that such important information be included in reference lists.

In  $\text{BIB}\text{T}\text{E}\text{X}$  title fields, braces protect words, such as proper nouns and acronyms, that cannot be downcased if the selected bibliography style would otherwise do so. In German, all nouns are capitalized, and the simple way to ensure their protection is to brace the entire German text in the title, as we did in the entry above.

The world's first significant computer program may have been that written in 1842

by Lady Augusta Ada Lovelace (1815–1852) for the computation of Bernoulli numbers [16, 18]. She was the assistant to Charles Babbage (1791–1871), and they are the world’s first computer programmers. The programming language *Ada* is named after her, and is defined in the ANSI/MIL-STD-1815A Standard; its number commemorates the year of her birth.

We do not discuss mathematical *transforms* in this dissertation, but you can find that phrase in the index (except that this sample thesis doesn’t have one!)

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- [11] ———, *Eine Beziehung zwischen dem elastischen Verhalten und der spezifischen Wärme bei festen Körpern mit einatomigem Molekül. (German) [A relationship between the elastic behavior and the specific heat of solid bodies with monatomic molecules]*, Annalen der Physik (1900) (series 4), 339 (1911), pp. 170–174, 590. See remarks [9, 10].

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