Wave Motion Visual Aids
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Introduction
Our goal was to create visual aids that can be used to assist teaching high school age students about Rayleigh, Transverse, and Longitudinal waves. Three separate mechanical devices were created that replicate the motion of each waveform. The characteristic motion of each wave type is unique.
• Transverse waves: Particles move perpendicular to the wave.
• Longitudinal waves: Particles move parallel to the to wave.
• Rayleigh waves: Particles move in elliptical trajectories.

Design Objectives
General objectives of the project were identified based on discussion with our advisor Dr. Kingstedt, and quantifiable test metrics were created to accomplish those objectives.

<table>
<thead>
<tr>
<th>Design Objective</th>
<th>Test Metric</th>
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<tbody>
<tr>
<td>Portable</td>
<td>Weight</td>
</tr>
<tr>
<td>Easy to use</td>
<td>Mechanical Resistance</td>
</tr>
<tr>
<td>Accurately depict wave motion</td>
<td>Motion Tracking and comparison to digital models</td>
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Mechanical Resistance Testing
Mechanical resistance was measured by applying a constant force (using a pulley and suspended weight) to each machine and measuring the time it took for the machine to move a certain distance. A linear fit was applied to the data to calculate the average resistance force, $F_R$.

$$F_R = \frac{\Delta x}{\Delta t^2}$$

$$\Delta x = \frac{F_W}{F_R - F_W} \times a$$

$$\Delta T = \frac{T_W - T_R}{J \dot{\theta}}$$

$\Delta x$ is the distance moved, $\Delta t$ is the time taken, $F_W$ is the weight, $F_R$ is the resistance force, $a$ is the acceleration, $T_W$ is the applied torque, $T_R$ is the resisting torque, $J$ is the moment of inertia, and $\dot{\theta}$ is the angular velocity.

Friction Test Data for Rayleigh Wave Machine

Model Accuracy
The accuracy of each machine can be determined by comparing the actual profile of each machine to the theoretical profile generated by a MATLAB model. The motion of each wave can then be measured by recording the motion with a camera and performing a digital image analysis. The integral of each wave profile could then be calculated and the error between the two compared.

$$\int_{t_1}^{t_2} \omega = \Delta \theta$$

$\omega$ is the average angular velocity, $\Delta \theta$ is the change in angular position, and $t_1$ and $t_2$ are the time intervals.

Equations of Motion
$$\Sigma F = F_W - F_R = ma$$

$$\Sigma T = T_W - T_R = J \dot{\theta}$$

Resistance Test Experimental Setup

Rayleigh Wave Machine uses a gear rack system to incorporate translational and rotational motion in the movement of the pins to create the desired particle trajectory.

Transverse Wave Machine uses a sliding internal wave form to push a series of visible rods to create the desired particle trajectory.

Longitudinal Wave Machine uses a series of mobile balls connected by wire that is driven by a camshaft to create the desired particle trajectory.

Conclusion
We encountered several issues during the construction and testing of these machines.
• The Transverse wave machine was never tested because the wave mold would induce a moment on the pins resulting in the pins jamming in the hole that guide them vertically.
• The Longitudinal wave had issues with constraining the wires along the proper path. Additionally, the cams are slightly uneven in their alignment. The camshaft was tested for resistance, but our testing strategy couldn’t measure a large enough resistance.
• The Rayleigh wave was operable; however, the motion tracking procedure was ineffective. The code was never able to successfully track the motion of all particles at once and would fail when attempting to.

Future Work
• Identify and solve rod jamming issue in Transverse Wave Machine.
• Rebuild Longitudinal machine camshaft so that cams are flush against one another. Redesign machine to use smaller diameter music wire, which is easier to work with.
• Improve or replace motion tracking program so model verification can be accomplished.

Results

<table>
<thead>
<tr>
<th>Description</th>
<th>Metric</th>
<th>Transverse</th>
<th>Longitudinal</th>
<th>Rayleigh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machine weight</td>
<td>&lt;10 lbs</td>
<td>8.3 lbs</td>
<td>6.0 lbs</td>
<td>5.6 lbs</td>
</tr>
<tr>
<td>Resistance</td>
<td>&lt; 2 lbs</td>
<td>Untested</td>
<td>&gt; 0.5lbs (0.1 lb inch)</td>
<td>0.12 lbs</td>
</tr>
<tr>
<td>Motion Error</td>
<td>&lt; 10%</td>
<td>Untested</td>
<td>Untested</td>
<td>Untested</td>
</tr>
</tbody>
</table>

Example of wave comparison. In Blue is the smooth theoretical wave. In Orange is the wave profile captured by the motion tracking. A smaller area between the two curves means a more accurate wave.